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Working Paper 01/18

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Fiscal sustainability vs. fiscal stability: tax and debt under entitlement spending*

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Abstract

Economists have traditionally used a rule that restricts primary deficits to less than a threshold determined by the interest-growth rate differential and existing debt in order to judge fiscal sustainability. This rule derives from a single period application of the government's budget constraint. It is not forward looking. It does not allow for the predictable dynamics of spending liabilities, such as entitlement spending, and assumes immediate and infinitely elastic tax or spending adjustments that are unlikely to be feasible in practice. To address this issue, we derive the equivalent dynamic rule: the primary surplus needs to match any expected discounted increases in public spending, the net interest on existing debt, and terms reflecting the cost of extending debt relative to changing taxes. We find strong and robust empirical evidence supporting the model and we calibrate our model to analyze the impact of shocks to future sustainability (as opposed to current stability, a crucial distinction) in different countries.

JEL classification: E62, H53, H63, I38 ;

Keywords: Sustainable public debt, primary deficit rules, fiscal space.

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1 Introduction

Households need to smooth consumption over time. But tax volatility reduces income stability and utility. Moreover, since taxes produce distortions and distortions increase as larger shares of income are taxed, tax smoothing is important for minimizing the misallocation of resources. Public debt, on the other hand, allows tax smoothing by reallocating the tax increases needed to fund expenditure shocks over time. Debt is therefore useful for financing transitory expenditure shocks.

But is debt also useful in the presence of *permanent* or long term expenditure shocks? This is important since entitlement spending represents a substantial share of public expenditure in all developed economies, and entitlement spending shocks are extremely persistent. The standard answer provided by economists is that the nature of the shocks is irrelevant, as long as the cost of debt is stable. But debt stability is not so easy to achieve when spending shocks are permanent, and tax capacity is finite. Governments therefore face an inter-temporal choice: to trade off the costs of raising taxes now against the (discounted) costs of higher debt and hence higher taxes later. In this world, Ricardian equivalence would not hold.

We argue that the standard analysis of these issues does not recognize in full the costs of debt. Changes in the stock of debt in this environment are in fact costly, hence costs and benefits of debt and taxes need to be valued together in a dynamic framework. Similarly, the standard procedure adopted to manage public debt is implicitly based on a static framework for policy intervention and hence it involves a feedback or multi-period open loop sequential control, since policy makers simply respond ex-post to realized outcomes. In this paper we stress that to operate fiscal rules successfully, we need to adopt closed loop control, a process based on forecasting forward looking data plus feedback adjustments for when new information becomes available.[1]

In a conceptually similar framework, Ghosh et al. (2013) and Checherita et al. (2014) have developed the idea of fiscal space defined as the difference between the current debt level and a debt limit, or debt target, beyond which debt will become unsustainable. In that environment governments need to take care not to exhaust their fiscal space, implying the presence of a debt threshold beyond which interest rates rise disproportionately, making the need to raise large sums from financial markets increasingly problematic; and eventually impossible because markets will anticipate a default. But this world is binary: either governments face linear costs on debt, or the costs they face rise so sharply that they loose access to debt finance. By contrast, we model an environment where governments can predict the variation in the overall cost of debt as the debt burden rises. Hence they can plan in advance the path of debt and taxes by internalizing any non-linear costs, and respond to any shock by using an optimal combination of debt and tax instruments.

To implement this idea we develop a dynamic version of Barro's (1979) model where stocks matter. Following Bohn (1990), we add a convex cost term for tax revenues in order to capture the distortionary and political costs of an excess tax burden, not to mention the cost of exceeding the economy's fiscal capacity, to deliver the necessary revenues. We then add convex costs in the stock of debt on the assumption that capital markets and other institutional arrangements in different countries will want to impose a burden (which can be weak or strong) on excessive debt.

Given this setup, we study *both* the *stability* of such a system and the deficit rules that are necessary to ensure that *sustainable* fiscal positions will be achieved in the long term. Stability requires that any temporary shock will

[1] Hughes Hallett and Jensen (2012) and European Council (2012) have argued that debt is superior to deficits as a target to ensure fiscal responsibility, since debt is a stock while deficits are a flow.

create disturbances that diminish over time, so that the economy and its fiscal balances gradually return to their equilibrium steady state positions. By contrast, sustainability means that future fiscal balances can be managed efficiently even in the presence of permanent shocks, changes in policy preferences, or structural changes in economy's performance as interest rates, output growth or inflation rates change. This paper studies the stability of debt and provides a rule that allows us to test for sustainability looking into the future.

Two main results emerge from this analysis:

a) A new solvency rule to guarantee sustainable fiscal balances, which is the appropriate test when entitlement spending innovations are persistent (Section 3.1). The new rule still requires a primary surplus large enough to offset the impact of any positive wedge between the nominal cost of debt and the rate of nominal GDP growth, scaled by the debt-to-GDP ratio. It also includes additional terms. In particular the rule requires the primary surplus to be large enough also to absorb any increase in entitlement spending.

b) An explicit recognition that entitlement spending introduces a tax now vs. tax later trade off, because the volatility of tax and debt revenues generates costs. The traditional rule, by contrast, assumes that taxes may be increased or spending decreased instantly and indefinitely at zero cost.

The second part of our work applies the model to analyze separately both the stability of debt over time and the current sustainability of debt in a series of different countries. To analyze stability, we estimate the model with panel data techniques, making use of data from the Public Finances in Modern History Database collected by Mauro et al. (2015) covering a large sample of countries for periods of sixty years or more. We find that the estimated coefficient capturing the non-linear debt dynamics is always significantly different from zero, across different samples of countries and different time periods. Hence, our modelling strategy involving a dynamic framework is strongly and robustly supported by the empirical evidence.

To evaluate the sustainability of the current debt position of different countries, we analyze calibrated versions of our model, fitted to observed data from leading OECD economies to obtain examples of the optimal responses of debt and taxes following expenditure or interest rate shocks similar to those observed in the past. We find that expenditure persistence is more relevant than the initial debt burden for defining the impulse response of debt and taxes following a shock. Debt is unsustainable when tax revenues are a large share of output, and expenditure levels are large and persistent.

Finally, an important caveat: We simplify the analysis by considering debt as a tool to smooth the impact of volatility in output and tax revenues, but we assume that this volatility is produced by exogenous shocks only. Hence we do not consider the role of debt as a tool to smooth the volatility induced by the business cycle and the resulting complicated feedbacks between policy tools and output responses. Nor do we take into account the role of monetary policy or the interactions between fiscal and monetary policy. The modelling of these interactions would require a more complicated framework that would obscure our main points. But it is important to stress that the same results would go through in a more complicated model of fiscal policy with feedbacks between policy tools and output responses, since in our context the sources of volatility are irrelevant. Hence our conclusions would remain valid in any more complicated environment. By contrast the recent literature on theoretical models of public debt uses a general equilibrium approach treat interest rates and income levels as endogenous variables. But those variables are effectively static and hence incomplete because debt stocks do not play an explicit role as a state variable. They therefore cannot explain debt dynamics or persistence properly; or the tax vs debt policy choice; or the different tests for stability and sustainability.[2]

The structure of the paper is the following: Section 2 sets out a distinction between stability and sustainability of debt. Section 3 presents a theoretical model for the optimal intertemporal choices of debt and taxation, together with the conditions for debt sustainability and our new rule for debt stabilization. Section 4 reports the results of the empirical analysis supporting the model. Section 5 then illustrates an application of the model to several OECD economies by means of calibrated solutions. Section 6 discusses the relevance of the results and concludes.

2 Stable Fiscal Balances vs. Sustainable Fiscal Policies

Several countries currently display historically high levels of debt that instead of being temporary spikes following extraordinary events such as wars, have now become permanent features of the economic environment. Debt levels have also been rather stable and persistent. But will they remain so in the future? To address this issue, we draw an important distinction between the stability of an economy's fiscal balances and the sustainability of its public financing. By the former we mean the dynamics of the fiscal system and economy underlying

[2]For example Davig et al. (2010, 2011) study the constraints that unfunded entitlement spending and policy uncertainty might impose on monetary policy, debt sustainability, tax revenues and interest costs, while the empirical work by Leeper et al. (2010) suggests that different tax instruments affect fiscal multipliers. In neither case do they supply a fiscal rule to ensure sustainable fiscal balances. Similarly, a significant share of the optimal Ramsey tax literature in dynamic models either rules debt out, or includes it but does not highlight its dynamic properties. The main exceptions—such as Chari and Kehoe (1999), Aiyagari et al. (2002), or Bhandari et al. (2016)—focus instead on the problems caused by the presence of incomplete markets, and do not model other institutional constraints on debt issuance.

it, inclusive of the current choice of fiscal policies, is stable in the conventional sense: that is, a one-off or temporary shock will create disturbances that diminish over time, so that the economy and its fiscal balances gradually return to their status quo ante (or equilibrium steady state) positions. But by contrast, sustainability means that, given stability (so that the current debt burden is not driven by explosive behavior), it is reasonable to expect future fiscal balances to be financeable without undue stress or a financial collapse when there are permanent shocks, changes in policy (or political) preferences, or structural changes in economy's performance (with respect to interest rates, output growth or possibly relative inflation rates for example).

On these definitions, our conventional notion of stability is a necessary condition for fiscal sustainability and means that the current debt burden is not explosive. But it is not sufficient. The ability to finance expected or likely future spending, deficits or debt, is the sufficient condition for testing fiscal sustainability. In Section 3.1 we provide a rule that allows us to test for sustainability looking into the future.

Why might sustainability fail? Two obvious examples are:

i) If entitlement spending rises either because there is a shift to a more generous system (an institutional change, or a change in political preferences), or because the coverage widens (baby boomers perhaps), or because the economy encounters bad times (a long-term negative shock). This is the case which is the subject of this paper.

ii) Fiscal policy preferences change: the penalties on failing to control fiscal deficits and debt are reduced, so that more expansive policies are permitted or encouraged for example. This could interfere with the stability condition in the future.

3 An intertemporal framework for debt and taxation

In this section we derive an intertemporal framework for governments that internalizes the costs that are expected to arise from raising revenues or taxes, under the assumption that a substantial share of expenditure consists of entitlement spending that can be treated as an exogenous stochastic process: a dynamic version of the of Barro's (1979) model. Our approach is based on the explicit recognition that policy reviews of government spending are generally carried out on the basis that there is a clear-cut difference between discretionary spending, and entitlement spending for which funding is mandatory unless explicit reform measures are proposed in Parliament. Entitlement spending therefore includes all those categories of expenditure that do not have to be approved annually since existing laws give agents rights to certain benefits or services until a wholesale reform is called. Two examples are social security/social benefits and the compensation of employees. By contrast, discretionary spending is composed of those expenditures that politicians have to vote annually or on an ad hoc basis in order for them to take place. Discretionary spending also includes categories of intermediate consumption that need to be voted annually in budgetary sessions even if they then recur for several years.[3]

Accordingly, defining all variables as ratios with respect to GDP, we assume that public expenditure G_t is composed of entitlement spending E_t and discretionary spending V_t, where entitlement spending is an exogenous stochastic process:

$$G_t = E_t + V_t. \tag{1}$$

By substituting the value of taxes (T_t) and expenditure (G_t) for primary deficit

[3]The share of discretionary spending is typically about 1/3 of total government spending in the OECD. See Coricelli and Fiorito (2013).

(D_t), we can express the law of motion of the debt ratio (B_t) as:

$$T_t = G_t - B_t + (1 + r_{t-1} - \rho_{t-1})B_{t-1}. \tag{2}$$

To keep the model tractable we assume that discretionary spending is financed with tax revenues (but borrowing is allowed to cover entitlement spending):

$$V_t = \tau T_t. \tag{3}$$

and thus

$$T_t = \psi\Big[E_t - \Delta B_t + (r_{t-1} - \rho_{t-1})B_{t-1}\Big]. \tag{4}$$

where $\frac{1}{1-\tau} = \psi$, and $0 < \tau < 1$. Since discretionary spending is smaller than entitlement spending, this assumption does not imply any restrictions. The choice of funding by taxation or debt is now made for a given or predicted composition of discretionary and entitlement spending.

This formulation highlights that any convex cost on T_t generates an adjustment cost on the stock of debt.[4] As a consequence, the problem becomes dynamic even without assuming non-linear cost functions for the stock of debt or its adjustment. Our modelling strategy therefore differs from the standard approach that analyzes the stock public debt as a sequence of period by period independent budget constraints, without recognizing the intertemporal constraints.

In our model, the objective is to provide the chosen expenditures while minimizing taxes and interest costs. The dynamic objective function includes

[4]The convexity assumption of Bohn (1990) is supported by the empirical results of Agell (1996) and Agell et al. (1996), suggesting that the distortionary effect of taxation are small for low levels of taxation, but they grow rapidly as taxes increase. These issue are extensively discussed by Leeper and Walker (2011).

both flows and stocks, and can be written as follows:

$$GOF = E_t + \zeta V_t + \frac{\delta}{2}(V_t)^2 - \nu T_t - \frac{\varphi}{2}(T_t)^2 - \frac{\phi}{2}(B_t)^2 - r_t B_t. \tag{5}$$

Here entitlement spending generates linear benefits normalized to one and discretionary spending generates linear and convex benefits whose importance is defined by the ζ and δ parameters. Similarly, ν and φ define linear and non-linear cost parameters associated with tax revenues. They include both resource costs induced by distortionary taxation, and political economy restrictions on government behavior induced by the voters' aversion to taxes. Finally, the servicing of debt imposes a standard interest cost r_t. But the government needs to internalize the non-linear cost ϕ, caused by non-linearly increasing risk premia in interest payments, or by the voters' aversion to public debt.

Policymakers then maximize (5), subject to (4) and equations that define the stochastic process of expenditure, interest rates, and GDP growth. For example, we can model the latter as exogenous AR(1) processes:

$$E_{t+1} = E_t + \alpha + \eta_{t+1}, \tag{6}$$

$$r_{t+1} = r_t + \beta + \epsilon_{t+1}, \tag{7}$$

$$\rho_{t+1} = \rho_t + \gamma + \theta_{t+1}, \tag{8}$$

where η_{t+1}, ϵ_{t+1}, and θ_{t+1}, are *i.i.d.* random shocks with zero mean and constant variances.

Defining d as the discount factor, the Lagrangian form of the problem is:

$$\Lambda = d^t \Big\{ E_t - \gamma T_t - \frac{\alpha}{2}(T_t)^2 - \frac{\phi}{2}(B_t)^2 - r_t B_t + \mu_t \Big[T_t - \psi E_t + \psi B_t - \psi(1 + r_{t-1} - \rho_{t-1}) B_{t-1} \Big] \Big\}, \tag{9}$$

where $\gamma = \nu - \zeta\tau$ and $\alpha = \varphi - \delta\tau^2$. The term in α represents the non-linear cost of taxes, net of benefits obtained from discretionary spending. Similarly, γ measures the linear cost of taxes net of benefits from discretionary spending.

Defining $\beta = \phi(1-\tau)$ and $\upsilon = 1-\tau$,[5] the first order conditions identify a law of motion for public debt that shows how expenditure shocks are absorbed by new debt issuance:[6]

$$\Delta B_{t+1} = (1 + r_t - \rho_t)G_{t+1} - \frac{1}{d}G_t - (r_t - \rho_t)B_{t+1} + [(r_t - \rho_t)^2 + 2(r_t - \rho_t)]B_t + \frac{1}{d}B_t - \frac{1}{d}(1 + r_{t-1} - \rho_{t-1})B_{t-1} + \frac{1}{d\alpha}\Big[- \gamma(1-d) + (\upsilon + d\gamma)r_t - d\gamma\rho_t + \beta B_t\Big]. \quad (10)$$

Both the response to new issues of debt, to the existing stock of bonds and market interest rates are functions of the relative weights of the non-linear cost of debt and taxes. For higher levels of the tax parameter α, the impact of interest rates and the past stock of debt declines. The impact of interest rates rises with the linear cost term γ, while the impact of the past stock of debt is large for larger values of β measuring the convex cost on the stock of debt.

This second order difference equation in B_{t+1} (which includes the debt implicit in the anticipated deficits D_{t+1} and D_t) cannot be solved with standard techniques proposed by Sargent (1979) because r_t is part of the intercept term and also of the slope. The roots in the solutions are thus time-varying.

[5] And thus $\phi < \beta$ and $\upsilon < 1$.

[6] See Cerniglia et al. (2018) for details. However, the model in this paper has a modified set of dynamics.

3.1 The new solvency rule

The law of motion derived in Equation (10) can be rewritten to make the role of primary deficits and surpluses explicit:

$$\Delta B_{t+1} = (1 + r_t - \rho_t)G_{t+1} - \frac{1}{d}G_t + (r_t - \rho_t)(1 + r_{t-1} - \rho_{t-1})B_{t-1}$$
$$-(r_t - \rho_t)\Delta D_{t+1} + \frac{1}{d}D_t + \frac{1}{d\alpha}\Big[- \gamma(1-d) + (\upsilon + d\gamma)r_t - d\gamma\rho_t + \beta B_t\Big], \quad (11)$$

which yields a rule which defines a primary *surplus* sufficient to stabilize debt:

$$-D_t = [d(1 + r_t - \rho_t)G_{t+1} - G_t] + d(r_t - \rho_t)(B_t - D_{t+1}) +$$
$$+\frac{1}{\alpha}\Big[- \gamma(1-d) + \upsilon r_t + d\gamma(r_t - \rho_t) + \beta B_t\Big]. \quad (12)$$

where $\upsilon = 1 - \tau$. This rule defines a threshold above which the primary surplus will reduce the debt ratio, and below which debt increases. It is however, quite different from the standard textbook rule which, in this notation, would have been $-D_t = (r_t - \rho_t)B_t$. In fact (12) contains three terms:

i) The first represents the discounted growth adjusted increase in total public spending relative to national income.

ii) The second is the counterpart to the standard textbook rule, but discounted and adjusted for the expected/planned surplus next period.

iii) The last term represents all the preference factors (and post-tax interest rate) that arise because we need to account for intertemporal optimizing behavior: that is, for the choice between raising taxes now or extending debt/taxing later to finance current or future spending liabilities. The trade-off between using taxes vs. debt is likely to dominate (12) since the remaining terms will be small compared to $(\beta/\alpha)B_t$.

The implication of (12) is that the old period-by-period criterion is no longer sufficient to ensure sustainable public finances once entitlement spending enters the story. Instead, we need to impose a condition that public spending shall not grow faster than national income by a certain margin and subtract the effect of preferring to tax now vs. increasing debt from expected growth.

When the policy-maker has a strong aversion to debt, so β is larger, or when interest costs are large, the primary surplus required will be larger. By contrast, the higher are the distortions caused by high tax revenues relative to income, the lower is the primary surplus required because debt becomes a more attractive tool to smooth any expenditure shocks.

3.2 The stability of debt

Debt stability is an important issue even in the absence of default. In our setting the expected instability of debt could make the intertemporal plans of the government not credible, and rational investors would not finance the debt. For the purposes of analyzing debt stability, Eq. (11) can be reformulated to give a form that both incorporates the cross-parameter restrictions implied by our theoretical model, and avoids the simplifying assumption that $r_t - \rho_t = \kappa$ is constant. To accomplish this, we rewrite Eq. (11) as:

$$\begin{aligned} \Delta B_{t+1} &= a_0 + a_1 G_{t+1} - a_2 G_t + a_3 B_{t-1} \\ &-a_4 \Delta D_{t+1} + a_5 D_t + a_6 B_t + a_7 r_t + a_8 \rho_t, \end{aligned} \tag{13}$$

where

$$a_0 = \frac{\gamma(1-d)}{d\alpha} \qquad a_1 = 1 + r_t - \rho_t \qquad a_2 = -\frac{1}{d}$$
$$a_3 = (r_t - \rho_t)(1 + r_{t-1} - \rho_{t-1}) \qquad a_4 = -(r_t - \rho_t) \qquad a_5 = \frac{1}{d} = a_2$$
$$a_6 = \frac{\beta}{d\alpha} \qquad a_7 = \frac{\upsilon + d\gamma}{d\alpha} \qquad a_8 = -\frac{\gamma}{\alpha}.$$

Equation (13) allows us to look at debt directly. The debt dynamics[7]

$$\{B_{t+1} - G_{t+1} - (r_t - \rho_t)[(1 + r_{t-1} - \rho_{t-1})B_{t-1}]\} =$$
$$a_0 + a_2 T_t + (1 + a_6)B_t + a_7 r_t - a_8 \rho_t + (r_t - \rho_t)[\Delta T_{t+1} + G_t] + \epsilon_t; \tag{14}$$

will be stable if and only if the fiscal system

$$\begin{bmatrix} B_{t+1} \\ B_t \end{bmatrix} = \begin{bmatrix} 1 + a_6 & c \\ 1 & 0 \end{bmatrix} \begin{bmatrix} B_t \\ B_{t-1} \end{bmatrix} + a_0 + a_2 T_t +$$
$$a_7 r_t - a_8 \rho_t + G_{t+1} + (r_t - \rho_t)[\Delta T_{t+1} + G_t] + \epsilon_t, \tag{15}$$

has roots inside the unit circle, where $c = (r_t - \rho_t)(1 + r_{t-1} - \rho_{t-1})$ and where T_t are choice variables at each t. The stability of the fiscal system is controlled by the roots of (15), $\lambda_{1,2}$, obtained by solving

$$\lambda^2 - (a_6 + 1)\lambda - c = 0, \tag{16}$$

where $a_6 > 0$ by definition given the policy cost function (5). Hence,

$$\lambda_{1,2} = \frac{1}{2}\{a_6 \pm \sqrt{(a_6+1)^2 + 4c}\} = \frac{a_6 + 1}{2} \pm \sqrt{\left(\frac{a_6+1}{2}\right)^2 + c}. \tag{17}$$

[7]Notice that the distinction sometimes drawn between global stability and saddle path stability does not apply here: a first order lead-lag (forward, backwards) dynamic system is equal to a renormalized second order (backwards) difference equation provided the coefficients on the lead term are invertible [Fisher and Hughes Hallett (1988)]. This is the case in (18).

Note that c will have the sign of $r_t - \rho_t$ in all but quite exceptional circumstances since $r_{t-1} - \rho_{t-1}$ will measure only a few percentage points and therefore be a good deal less than unity. That means there are now just two cases to consider:

1. $r_t - \rho_t > 0$, so $c > 0$. In this case, the dynamics implied by (10) and (15) are always unstable since (17) implies $\lambda_1 > 1 + a_6 > 1$. So at least one root greater than unity, implying instability and unsustainable fiscal policies.

2. $r_t - \rho_t < 0$, so $c < 0$. The positive root from (17), λ_1 will be in the unit circle if

$$\sqrt{\left(\frac{a_6+1}{2}\right)^2 + c} < 1 - \frac{a_6+1}{2} \qquad \text{that is, if} \quad c < -a_6. \tag{18}$$

In addition, we need to have the right hand side positive: $1 - \frac{a_6+1}{2} > 0$. This requires $a_6 < 1$, or $\beta < d\alpha$, given the definition of a_6. This is the necessary condition; the rest of the inequality, $c < -a_6$, is sufficient. This restriction is interesting since, with little or no discounting ($d \simeq 1$), it implies that policymakers have to worry more about taxes than debt. Intuitively: if I don't discount, I have to force taxes to be consistent with stability. The debt that then follows will be stable. But if I do discount, I can loosen taxes to some extent because the mild debt instability (and tax increases) that follow will be discounted away. However when the reverse inequality holds, the stability condition can never be satisfied.

Conversely, with full discounting ($d = 0$), the only thing that matters is the debt level (the term in γ, since $\beta \leq 0$ would then be necessary). This is the traditional primary surplus rule. But in this case a_6 is infinite, so the system's roots are infinity and zero, and its dynamics have become a random walk. In that case, the system is unstable unless policies are set to exactly offset each of the shocks as they arise— as in the traditional period-by-period primary surplus rule. This is the result that makes the case for this paper: you must plan for the future if you want to maintain debt stability under entitlement spending, tax smoothing, or persistence in the deficit and other forms of fiscal dynamics.

Finally, notice that the negative root in (17), λ_2, is in the unit circle if $c < a_6 + 2$. This condition is automatically satisfied since the right hand side is positive.

Comments:

1. It is worth noting that stability in this type of model involves roots that are based on time-varying parameters in the equations of motion: (10), (11) and hence policy rule (12). To be fair, this was also the case in the standard rule $-D_t = (r_t - \rho_t)B_t$, but is seldom remarked upon as the latter is derived for one period at a time. This means that, if you operate a fixed rule, you may have periods of temporary instability requiring periods of greater stabilization to keep the economy on a stable path overall.

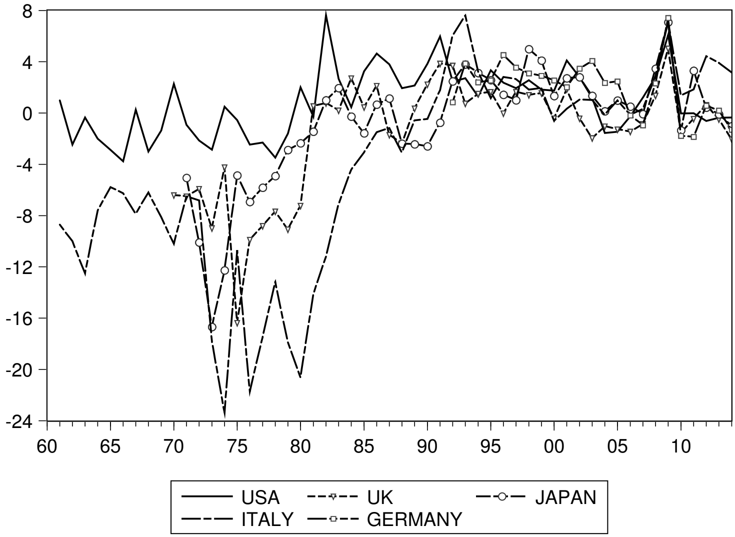
2. In the stable case, $r_t - \rho_t < 0$, the necessary condition to have positive debt in steady state, which is $\frac{\beta}{\alpha} < \kappa[1 - d(1+\kappa)]$, can be satisfied even if $\frac{\beta}{\alpha} < 0$.[8] The model can therefore have a stable equilibrium even if the convex cost of debt is zero or less because the convex cost on taxes generates an adjustment cost on the stock of debt. Hence debt will have a natural ceiling in steady-state if there are costs to using the tax instrument. Put differently, Ricardian equivalence does not hold here because both higher debt now and higher taxes in the future generate substantial costs. It therefore pays to restrain debt earlier rather than later.

[8]Steady state equilibrium values of the model are discussed in Appendix I.

3. In the unstable case, it is interesting to analyze when the Blanchard-Kahn conditions apply, since these conditions define a saddle-path equilibrium, and in this case the system remains controllable. The Blanchard-Kahn conditions require that $\kappa + \frac{1}{d(1+\kappa)} + \frac{\beta}{d\alpha(1+\kappa)} > \frac{1}{d}$. In the limiting case of no depreciation and $\kappa = 0$, the condition becomes $\frac{\beta}{\alpha} > 0$. In general, a saddle-path equilibrium requires a positive value of $\frac{\beta}{\alpha}$ for values of κ in the range $0 \leq \kappa \leq \frac{1-d}{d} = k^{**}$, negative otherwise.

Has this stability condition worked in practice? Figure 1 displays the value of $\kappa_t = r_t - \rho_t$ for several countries over the 1960-2014 period. While there is substantial variability across countries and time, it is clear that the values of κ_t were substantially negative during the first decades of the sample, but positive in the second part. As a consequence, debt has been stable during the first half-sample in most countries, but has become unstable in recent decades.

Figure 1: Values of κ for several countries over the 1960-2014 period.



4 An empirical assessment of the impact of institutional constraints on debt dynamics

4.1 Estimating equation

The aim of the empirical analysis of this section is to provide evidence that our dynamic specification of the problem is supported by the data. Hence we estimate the empirical counterpart of Eq. 13, where our parameter of interest is a_6. Our model is empirically supported if, and only if $a_6 \neq 0$. The estimating equation would then be

$$X_{t+1} = b_0 + b_1 T_t + b_2 r_t + b_3 \rho_t + b_4 B_t, \tag{19}$$

where

$$X_{t+1} = \{\Delta B_{t+1} - G_{t+1} - (r_t - \rho_t)[(1 + r_{t-1} - \rho_{t-1})B_{t-1}]\} \\ + (r_t - \rho_t)[\Delta T_{t+1} + G_t]\,, \tag{20}$$

and where $b_4 = a_6 = \frac{\beta}{d\alpha}$.

4.2 Estimation techniques

Before estimating the model we address the issue of the presence of cross-sectional dependence in our data set. Since we are analysing the behaviour of fiscal variables in economies that are interdependent, it is likely that the countries in our sample were affected by common shocks during this period of time and this can cause cross-sectional correlation in the residuals (Chudik and Pesaran (2013)). The countries in our sample compete for the same market when issuing debt instruments; they are trading partners and as such face similar business, fiscal, and financial cycles. When testing for cross-sectional dependence, the hypothesis of cross-sectional independence is strongly rejected in all cases, both for the entire sample and for the subsamples we consider. Thus, to address the cross-sectional dependence in our panel, we run fixed effects estimation and we use Driscoll-Kraay standard errors in our analysis.

4.3 Data

The fiscal data come from the Public Finances in Modern History Database collected by Mauro et al. (2015). To our knowledge this is the most comprehensive database on public finance available. We focus on developed and developing countries for which a time series of at least fifty data points is available.[9] All

[9] The initial year of the data for each of the countries is the following: United States 1800, United Kingdom 1830, Italy 1862, Canada 1870, Australia 1913, Portugal 1919, Mexico 1925, Switzerland 1929, Ireland 1936, Spain 1940, France 1946, Norway 1946, Sweden 1946, Japan 1947, New Zealand 1947, Austria 1948, Finland 1948, Greece 1948, Belgium 1949, Netherlands 1949, Germany 1950, Iceland 1951, Denmark 1954.

the data correspond to annual observations and all variables are expressed as ratios to GDP. We analyze both an unbalanced panel exploiting all the possible datapoints, and balanced panels including different samples of countries, to make sure that the results are robust to variations in the composition of the sample of countries.

To allow for the possibility that policy and behaviour with respect to debt management, taxation, public spending and entitlement spending have shifted over time, we split the sample into separate periods to provide individual estimates of our model for the longest available sample covering the period 1800-2011, or a shorter 1800-1989 sample that excludes the years of the great moderation. We also analyze a balanced panel covering 23 countries for both the complete sample 1954-2011, and a shorter 1954-2007 sample excluding the years of the financial crisis.

In line with the available empirical evidence discussed in footnote 3, we assume a share of entitlement spending $\nu = 0.7$ in the calculations.

4.4 Results

Table 1: Estimates of Eq. 19

	1800-2011	1800-1989	1954-2011	1954-2007
T_{t-1}	-0.92***	-1.00***	-0.82***	-0.87***
	(0.02)	(0.03)	(0.03)	(0.02)
B_{t-1}	-0.04***	-0.06***	-0.04***	-0.03***
	(0.01)	(0.02)	(0.01)	(0.01)
r_{t-1}	-0.40**	0.23	-0.70***	-0.65***
	(0.18)	(0.28)	(0.17)	(0.15)
ρ_{t-1}	0.48***	0.46***	0.52***	0.38***
	(0.09)	(0.10)	(0.10)	(0.06)
Constant	-0.67	0.39	-3.66***	-2.25***
	(0.71)	(0.83)	(1.01)	(0.75)
Observations	2305	1638	1524	1236
Groups	32	26	32	23
R^2	0.82	0.76	0.82	0.88

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 1 displays the results for the whole set of countries. The estimated coefficient $\hat{b}_4$ is always different from zero in all different samples and time periods, and the parameter is significant at the one percent level. The estimated parameter values for all other variables are similarly significant and the signs are all in line with the predictions of the model.

Tables 2 and 3 repeat the analysis after regrouping the countries in two sample of, respectively, low and high debt countries, where the low-debt countries are those whose debt-to-GDP ratio has never risen above 90% in our sample period. The results for the two classes of countries are in line with those for the whole sample. The main difference between the two sets of results is that in the

Table 2: Estimates of Eq. 19, low debt countries

	1800-2011	1800-1989	1954-2011	1954-2007
T_{t-1}	-0.94***	-1.00***	-0.85***	-0.86***
	(0.02)	(0.03)	(0.02)	(0.02)
B_{t-1}	-0.05***	-0.06***	-0.04***	-0.04***
	(0.02)	(0.02)	(0.01)	(0.01)
r_{t-1}	-0.23	0.20	-0.46**	-0.49**
	(0.18)	(0.29)	(0.18)	(0.22)
ρ_{t-1}	0.32***	0.27***	0.48***	0.41***
	(0.07)	(0.07)	(0.10)	(0.09)
Constant	0.44	1.57*	-2.68***	-2.48***
	(0.61)	(0.83)	(0.91)	(0.89)
Observations	1359	927	962	750
Groups	21	16	21	14
R^2	0.85	0.81	0.83	0.87

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

case of high debt countries the $\hat{b}_4$ parameter is smaller and often not statistically significant, while in the case of low debt countries the estimated coefficient is larger and extremely significant. This suggests that the high debt countries have not been constrained in their use of debt, since the non-linear cost term they have faced is small. Debt constraints have instead efficiently limited low debt countries.

Another significant difference between the two groups of countries is the size of the impact of interest rate costs on the dependent variable (a composite variable closely but inversely correlated with tax revenues). The impact of interest costs is larger for high debt countries than for low debt ones. Low debt countries are effectively constrained by their debt-aversion, and their decision making is consequently less constrained by interest rates. High debt ones use debt more freely, but have to raise more tax revenues because of interest costs.

Table 3: Estimates of Eq. 19 high debt countries

	1800-2011	1800-1989	1954-2011	1954-2007
T_{t-1}	-0.89***	-1.02***	-0.70***	-0.90***
	(0.04)	(0.03)	(0.07)	(0.04)
B_{t-1}	-0.02**	-0.05**	-0.02*	-0.01
	(0.01)	(0.02)	(0.01)	(0.01)
r_{t-1}	-0.73**	0.35	-1.25***	-0.79***
	(0.29)	(0.39)	(0.25)	(0.15)
ρ_{t-1}	0.62***	0.61***	0.58***	0.37***
	(0.14)	(0.15)	(0.13)	(0.07)
Constant	-2.11*	-0.87	-6.88***	-1.80*
	(1.09)	(1.16)	(1.85)	(1.02)
Observations	946	711	562	486
Groups	11	10	11	9
R^2	0.78	0.71	0.81	0.90

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The finding that $\hat{b}_4$ is always different from zero is a necessary but not sufficient condition for the underlying parameter β to be significantly different from zero. To prove that a dynamic specification is required in this framework, we need to provide evidence that $\hat{b}_4$ is significantly different from zero. To do that, we need to recover the expected values and variances of the underlying parameters given that $\hat{\beta} = \frac{\hat{b}_4\hat{d}}{\hat{\alpha}}$ and that $\hat{\alpha} = \frac{-\nu\hat{b}_1}{\hat{b_2}+\hat{b}_3}$. That is to say, we need to find the first two moment of variables that are ratios among stochastic variables. to perform the necessary significance tests. This can be done using results from

Mood et al. (1974):

$$E\hat{\alpha} = -\frac{\nu E\hat{b}_1}{E\hat{b}_2 + E\hat{b}_3} - \frac{\nu E\hat{b}_1}{[E\hat{b}_2 + E\hat{b}_3]^3} Var(\hat{b}_2 + \hat{b}_3), \tag{21}$$

$$Var(\hat{\alpha}) \approx \frac{\nu E\hat{b}_1}{E\hat{b}_2 + E\hat{b}_3}\Big\{1 + \frac{Var(\hat{b}_2) + Var(\hat{b}_3)}{[E\hat{b}_2 + E\hat{b}_3]^3}\Big\}, \tag{22}$$

$$E\hat{\beta} = \frac{E\hat{b}_4}{E(\hat{\alpha}\hat{b}_1)} + \frac{E\hat{b}_4}{(E\hat{\alpha}\hat{b}_1)^3} Var(\hat{\alpha}\hat{b}_1), \tag{23}$$

$$Var(\hat{\beta}) \approx \Big(\frac{E\hat{b}_4}{E\hat{\alpha}E\hat{b}_1}\Big)^2 \left\{ \frac{Var(\hat{b}_4)}{(E\hat{b}_4)^2} + \frac{(E\hat{b}_1)^2 Var(\hat{\alpha}) + (E\hat{\alpha})^2 Var(\hat{b}_1) + Var(\hat{\alpha})Var(\hat{b}_1)}{[E\hat{\alpha}E\hat{b}_1]^2} \right\}. \tag{24}$$

Under the simplifying assumption that $COV(d, \beta) = 0$ and assuming a value $\nu = 0.7$.[10]

Tables 4, 5 and 6 display the expected values, standard errors and t-ratios of α and β using the estimated parameters all countries sample, as well as low debt and high debt samples. In the all countries sample, the expected values of both α and β are significantly different from zero in each of the different time intervals and all subsamples of countries. The results are similar but stronger for the low debt countries, as all values are more strongly significant — at least for the past 70 years. This suggest that all countries, and low debt ones in particular, have actually been constrained in their fiscal choices by the non-linear costs for taxes and debt for more than two-thirds of a century. High debt countries, on the contrary, were less tightly constrained in their management of debt, although in two of the four samples the expected value of β is statistically significant also for these countries also.

Debt stability requires the condition $r_t - \rho_t < 0$. With low GDP growth rates it becomes progressively more difficult to satisfy this condition — except

[10]In this case, given two stochastic variables X and Y, $E\Big(\frac{X}{Y}\Big) = \frac{EX}{EY} - \frac{1}{(EY)^2}COV(X,Y) + \frac{EX}{(EY)^3}Var(Y)$ and $var\Big(\frac{X}{Y}\Big) \approx \Big(\frac{EX}{EY}\Big)^2\big(\frac{Var[X]}{\mu_X^2} + \frac{Var[Y]}{\mu_Y^2} - \frac{2COV(X,Y)}{\mu_X^2\mu_Y^2}\Big)$.

Table 4: Statistical Significance of β Parameter Values, all countries

	1800-2011	1800-1989	1954-2011	1954-2007
$E\hat{\alpha}$	58.99	1.2023	-7.0175	-3.4710
$SE\hat{\alpha}$	20.325	0.4380	3.494	1.5305
$t-value$	(2.90)	(2.75)	(2.01)	(2.27)
$E\hat{\beta}$	0.00079	0.0570	-0.00794	-0.01184
$SE\hat{\beta}$	0.00026	0.02514	0.00334	0.005478
$t-value$	(3.04)	(2.27)	(2.31)	(2.16)

Notes: A share of entitlement spending $\nu = 0.7$ is assumed in the calculations.

Table 5: Statistical Significance of β Parameter Values, low debt countries

	1800-2011	1800-1989	1954-2011	1954-2007
$E\hat{\alpha}$	40.977	14.029	3183.25	-73.958
$SE\hat{\alpha}$	15.689	6.3545	306.296	22.359
$t-value$	(2.61)	(2.21)	(10.4)	(3.31)
$E\hat{\beta}$	0.03862	0.00442	0.00015	0.01608
$SE\hat{\beta}$	0.00074	0.00078	0.0000032	0.00068
$t-value$	(53.4)	(5.67)	(4.69)	(23.6)

Notes: A share of entitlement spending $\nu = 0.7$ is assumed in the calculations.

perhaps before and after the recent financial crisis when interest rates (real or nominal) fell to near zero while growth remained positive except in the dark years of the crisis itself (2009-11). This may explain the results of Auerbach and Gorodnichenko (2017), that borrowing in that period has tended not to increase indebtedness or borrowing costs, and why debt burdens appear to have eased following the crisis. Table 7 displays the calculated values for the ratio $E\hat{\beta}/E\hat{\alpha}$. These ratios, as discussed in Section 3.2, provide the institutional constraints on debt. As we can see from Table 7, they were always very close to zero in all samples, including in recent decades. By contrast, these numbers highlight that, in the older 1800-1989 sample, the institutional constraints made it more difficult for countries to hold debt in steady state, and virtually impossible for the current high debt countries to do so.[11] In fact, the high debt countries these institutional constraints have always been very weak, and have been entirely absent in recent decades.

Table 6: Statistical Significance of β Parameter Values, high debt countries

	1800-2011	1800-1989	1954-2011	1954-2007
$E\hat{\alpha}$	-54.2056	0.8848	-0.8607	-1.7330
$SE\hat{\alpha}$	16.58	0.3245	0.0951	0.3539
$t - value$	(3.27)	(2.73)	(9.05)	(4.89)
$E\hat{\beta}$	-0.0004546	0.629	0.03246	-0.0067
$SE\hat{\beta}$	0.00024	0.0301	0.00334	0.00656
$t - value$	(1.89)	(2.09)	(9.71)	(1.02)

Notes: A share of entitlement spending $\nu = 0.7$ is assumed in the calculations.

Table 7: Values of the ratios $E\hat{\beta}/E\hat{\alpha}$.

$E\hat{\beta}/E\hat{\alpha}$	1800-2011	1800-1989	1954-2011	1954-2007
Full sample	0.00001	0.047	0.0011	0.0034
High debt countries	0.0009	0.0003	0.00000005	-0.0000002
Low debt countries	0.000008	0.711	-0.00377	-0.0039

Notes: A share of entitlement spending $\nu = 0.8$ is assumed in the calculations.

[11] Cerniglia et al. (2018) show how to recover the underlying parameter values for fiscal preferences and the implicit targets under the assumption that governments manage their liabilities by following this inter-temporal framework.

5 Debt sustainability in the OECD economies

We now apply our model to analyze fiscal sustainability in different set of countries with shorter samples of data. To do that, we calibrate the model as if the observed level of the variables in the year 2014 was the steady state value resulting from the model.[12] We then generate impulse response functions following shocks to the exogenous variables of a magnitude similar to those observed in recent decades. These impulse responses allow us to analyze the impact of the shocks and size of the policy adjustment required given the current structural debt levels.

We do this using a constant value of κ and assuming parameter values that respect the Blanchard-Kahn conditions. Then, to check that our results are not specific to US data, or the time period studied, we calibrate the model for a subsample of five countries, United States, United Kingdom, Japan, Germany, Italy.[13] For the purposes of illustration, we start from the levels of expenditure, debt and interest payments in 2014, and calibrate the model assuming κ equal to the average value that we observe in the second part of the sample. That is closer to the most recent values than an average taken over the sample as a whole. Then, to evaluate the performance of the model, we leave the steady-state value of taxes free, and compare the values of taxes predicted by the model for each country with the values observed in the data for the year 2014. These values are displayed in Table 8.

For each country we estimate the stochastic processes of the exogenous variables, interest payments and the expenditure to GDP ratio ((6) and (7) respec-

[12] These steady-values are derived in Appendix I.

[13] The data on debt for Europe are those based on the Maastricht criterion provided by the OECD, while those of USA and Japan are from the IMF WEO. All other data are taken from the OECD's Economic Outlook Database, version EO96. A detailed description of the data is available at http://www.oecd.org/eco/outlook/Database_Inventory.pdf. Following Henning Bohn, we use a measure of gross interest payments rather than the commonly reported measure of net interest payments [See Bohn (2011)].

tively) as the AR(1) processes also displayed in Table 8. In these calibrations we use the value of the coefficient of persistence obtained from those estimates, while we choose the value of the intercept of the stochastic processes in order to match in steady-state the value of yearly interest payments and expenditures in 2014. We match the historical volatility of the variables by using the time-series standard deviations of the variables.

We impose some parameters symmetrically for all countries. In particular, γ is set at 0.5. This is in line with the results of Barro and Redlick (2011) who found a value of 0.54 from the estimated parameters of a linear regression designed to determine the average marginal tax rate from GDP growth using United States data for the period 1950-2006. This identifies the linear cost of varying taxes in terms of the consequent losses in output growth. Barro and Redlick (2011) also calculate an income multiplier of -1.1 with respect to tax revenues, which is compatible with convex costs for taxes and suggests a value of $\alpha = 0.1$. We then choose $\beta = 0.01$ because it implies a ratio of stock-to-flow costs of $\frac{\beta}{\alpha} = 0.1$, a reasonable value in light of the evidence of Table 7. Finally, we choose an *ad hoc* value for the discount factor necessary to match the observed values of expenditure and debt in 2014 to steady state values of the model.

The steady-state variable values and calibrated parameter values for each country are displayed in Table 9 and they are compared to the data for 2014. We observe that the predicted values for taxes are very close to the observed values in most countries; with the partial exception of Japan and United Kingdom, where the steady-state level of taxes is above the current outcome indicating that, to stabilize debt at its current level, both countries require a substantial increase in the level of taxation.

Next we obtain a set of impulse responses for each country individually, fol-

Table 8: Properties of selected variables for the United States of America, Japan, Germany, United Kingdom and Italy

Variable	μ	μ 1960-1987	μ 1988-2014	σ	$C.V.$
		United States of America			
Public Expenditure	0.314	0.299	0.331	0.03	0.09
Interest payments	0.074	0.082	0.066	0.024	0.32
κ	0.008	-0.0015	0.018	0.026	3.25
Estimated Ar(1) processes			Stationarity		
$E_t = 0.030 + 0.909 * E_{t-1}$			I(0)		
$r_t = 0.00019 + 0.994 * r_t$			I(1)		
		Japan			
Variable	μ	μ 1960-1987	μ 1988-2014	σ	$C.V.$
Public Expenditure	0.298	0.254	0.343	0.06	0.20
Interest payments	0.037	0.056	0.026	0.020	0.54
κ	-0.008	-0.041	0.014	0.045	5.96
Estimated Ar(1) processes			Stationarity		
$E_t = 0.010 + 0.979 * E_{t-1}$			I(0)		
$r_t = -0.00068 + 0.993 * r_t$			I(0)		
		Germany			
Variable	μ	μ 1960-1987	μ 1988-2014	σ	$C.V.$
Public Expenditure	0.437	–	0.437	0.02	0.05
Interest payments	0.048	–	0.048	0.016	0.33
κ	0.020	–	0.020	0.023	1.15
Estimated Ar(1) processes			Stationarity		
$E_t = 0.26 + 0.415 * E_{t-1}$			I(0)		
$r_t = -0.00167 + 0.991 * r_t$			I(0)		
		United Kingdom			
Variable	μ	μ 1960-1987	μ 1988-2014	σ	$C.V.$
Public Expenditure	0.386	0.387	0.385	0.03	0.08
Interest payments	0.069	0.086	0.057	0.026	0.37
κ	-0.016	-0.048	0.005	0.044	2.75
Estimated Ar(1) processes			Stationarity		
$E_t = 0.075 + 0.810 * E_{t-1}$			I(1)		
$r_t = 0.00057 + 0.928 * r_t$			I(1)		
		Italy			
Variable	μ	μ 1960-1987	μ 1988-2014	σ	$C.V.$
Public Expenditure	0.382	0.340	0.426	0.06	0.15
Interest payments	0.052	0.042	0.062	0.028	0.55
κ	-0.043	-0.144	0.019	0.078	1.80
Estimated Ar(1) processes	Stationarity				
$E_t = 0.019 + 0.960 * E_{t-1}$			I(1)		
$r_t = 0.00178 + 0.968 * r_t$			I(1)		

Notes: Sample: 1960-2014. Analysis of stationarity conducted by means of Advanced Dickey Fuller and Phillips-Perron unit root tests. μ is the mean value, σ the standard deviation, and $C.V.$ the coefficient of variation of the variables.

Table 9: Steady state values for the United States of America, Japan, Germany, United Kingdom and Italy

			United States of America		
Parameter values	$\gamma = 0.5$	$\alpha = 0.01$	$\beta = 0.001$	$\kappa = 0.020$	$d = 0.856$
Model steady state values		T=0.349	B=1.047	E=0.347	r=0.037
Data values		T=0.333	B=1.047	E=0.347	r=0.037
			Japan		
Parameter values	$\gamma = 0.5$	$\alpha = 0.01$	$\beta = 0.001$	$\kappa = 0.020$	$d = 0.977$
Model steady state values		T=0.406	B=2.464	E=0.401	r=0.009
Data values		T=0.334	B=2.464	E=0.401	r=0.009
			Germany		
Parameter values	$\gamma = 0.5$	$\alpha = 0.01$	$\beta = 0.001$	$\kappa = 0.020$	$d = 0.940$
Model steady state values		T=0.438	B=0.744	E=0.423	r=0.022
Data values		T=0.439	B=0.744	E=0.423	r=0.022
			United Kingdom		
Parameter values	$\gamma = 0.5$	$\alpha = 0.01$	$\beta = 0.001$	$\kappa = 0.020$	$d = 0.942$
Model steady state values		T=0.416	B=0.878	E=0.412	r=0.028
Data values		T=0.380	B=0.878	E=0.412	r=0.028
			Italy		
Parameter values	$\gamma = 0.5$	$\alpha = 0.01$	$\beta = 0.001$	$\kappa = 0.019$	$d = 0.921$
Model steady state values		T=0.488	B=1.306	E=0.463	r=0.032
Data values		T=0.476	B=1.306	E=0.463	r=0.032

lowing a one standard deviation shock to interest payments and the expenditure to GDP ratio. The United States, the United Kingdom and Germany have similar standard deviations of expenditure of 2 to 3 percent; while in Japan and Italy the standard deviation is 6 percent. Figures 2, 3, and 4 now display the responses following expenditure shocks for all the countries of the sample; and that for interest payments in one of them (the representative country, since the pattern in the latter case is very similar in all countries).

The impulse responses following an expenditure shock have a similar pattern in all countries, in that taxes and debt both display a hump-shaped reaction. That said, the responses of taxes and debt themselves are quite different from one another. Taxes rise substantially at impact, and reach a peak after a few periods. The response of debt, by contrast, has a peak after several periods and declines more slowly thereafter and follows a rather different pattern across different countries. In particular, in the United States, the shock generates a peak in debt of 10 percent after ten years, but it declines slowly thereafter to reach a net increase of roughly 5 percent after 40 years. Taxes by contrast rise rapidly and peak after five years; but, as for debt, half of the increase is still in play 40 years after the shock. The scale of the impact on taxes, however, is smaller—roughly one tenth of that for debt, with a peak impact of just 1 percent.

The shape of the response in the other countries is similar to that in the United States, but there are noteworthy differences. In Japan the peak impact on debt, of a shock twice as large as that in the US, is only fifty percent larger. Furthermore, the debt peak occurs after 20 years instead of 10. Moreover, the shock is far more persistent and hardly dies out. The impact on taxes, on the other hand, is larger than in the US; taxes almost immediately rise by four percentage points and then decline very slowly afterwards. Thus, notwithstanding near zero interest costs in Japan, the capacity of the country to use debt as a tool to smooth expenditure shocks is limited because the impact of spending shocks falls predominantly on taxes.

In Germany, the impact of the shock is smaller and less persistent; a 1.5 percent increase in spending raises debt in the same proportion on impact, with a peak soon after at 2 percent, and it dies out relatively quickly thereafter. Taxes rise very little: the initial impact is just above 0.2 percent, and it too declines immediately afterwards. Also, although Germany has a relatively high level of expenditure, spending is less persistent than in other countries so that the impact of a spending shocks is temporary and more efficiently dealt with by managing debt. Contrast the United States and Japan, where debt is a less efficient tool to manage expenditure shocks since the latter are highly persistent and debt levels are higher.

The impact of the shock in the United Kingdom is similar to that in Germany, albeit to a lower degree, since the shock can be efficiently absorbed by debt and because expenditure changes are not so persistent.

By contrast, in Italy debt can only absorb fifty percent of the shock. As a result there is a larger impact on taxes and a larger increase in debt. In fact, after 20 years, debt rises by 20 percent of GDP despite an expenditure shock of 6 percent and a tax increase of 3 percent. Moreover, the impact on both taxes and debt is extremely persistent, almost permanent, because the spending changes are so persistent.

Turning now to interest payments, the impact of a one standard deviation shock to interest payments is displayed for Italy, but the pattern is very similar in all the other countries and is not reproduced in each case individually.

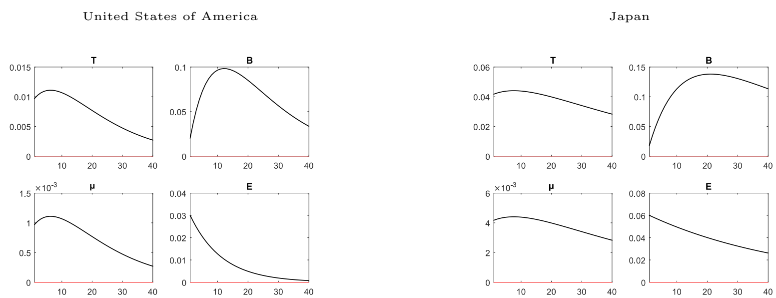


Figure 2: Impulse response functions following a one standard deviation shock to expenditure (η) in the United States of America and Japan.

Here taxes rise immediately to absorb the shock, and then decline monotonically therafter. Debt instead declines substantially after a hump-shaped response that peaks after several years. The peculiarity of the results is the scale of the impact: the changes in taxes and debt are several times larger than the changes in GDP, because the steady state level of debt is extremely sensitive to interest rate variations, particularly when small values of α are involved. This makes the volatility of debt less costly to the policymaker. Debt is also less sensitive to expenditure shocks, because the variable E is divided but also multiplied by a set of parameters of a similar magnitude that do not include α. Also for positive values of κ, however small, the steady state equilibria are fragile and especially sensitive to interest costs variations. In light of the level and persistence of expenditure, current levels of debt could therefore quickly become unsustainable in a higher interest rates environment.

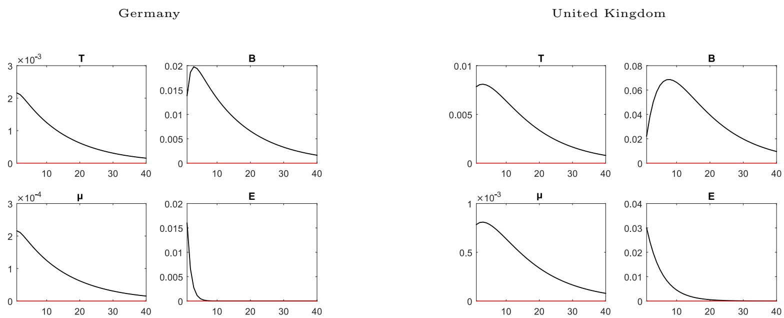


Figure 3: Impulse response functions following a one standard deviation shock to expenditure (η) in Germany, and the United Kingdom.

The implication of this analysis is that, although it is perfectly rational for a policymaker to cumulate a substantial amount of debt when κ is negative, particularly when κ is substantially negative as was the case in Italy, the equilibrium outcomes become fragile if κ turns zero or positive because of a shift to a low inflation environment. Hence disinflation must be matched by fiscal discipline to keep debt sustainable. That restricts our ability to use debt as a tool to absorb expenditure shocks — especially when those shocks are persistent.

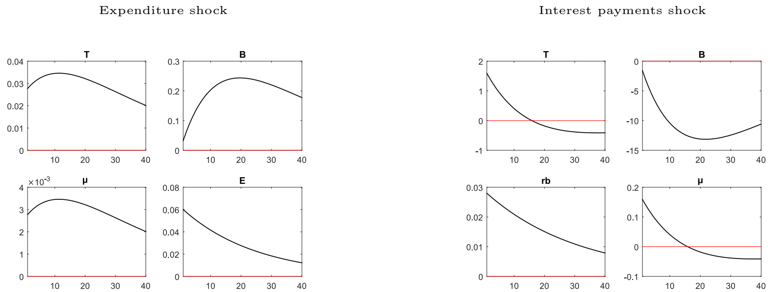


Figure 4: Impulse response functions following a one standard deviation shock to expenditure (η) and interest payments ϵ in Italy.

6 Conclusion

This paper has analyzed the optimal use of fiscal policy instruments in a world where there are entitlement spending liabilities. To do so involves moving from a static formulation based on the government's current period budget constraint, to an intertemporal rule since policymakers have to trade-off using taxes now to fund current and future spending vs. using debt, hence future taxes, to fund that spending.

We have derived a primary surplus rule to ensure debt stability in a dynamic world where there are entitlement spending liabilities, which is an extension of the standard-textbook role for debt stabilization to an environment where entitlement spending is predictable and the government internalizes any cost induced by the use of public debt. In this context the primary surplus needs to at least match any expected discounted increases in public spending, the net interest on existing debt, and terms reflecting the preference for extending debt relative to the cost of changing taxes. When policymakers have a strong aversion to debt (β is large), or if interest costs adjusted for growth are large, the primary surplus required is larger. But if the distortions due to high taxes relative to income are large, a lower primary surplus is required since debt becomes a more attractive tool to smooth spending shocks.

We have estimated the model using panel data on a large sample of countries, finding strong evidence that the non-linear debt cost parameters that support a dynamic specification of our problem are strongly significant.

Finally, we analyze calibrated versions of our model, fitted to observed data from a number of leading OECD economies, in order to examine the sustainability of debt and the optimal fiscal responses in those countries following expenditure or interest rate shocks. The outcomes for our rule calibrated to the OECD and larger national economies are substantially different from country to country and we find that the size of the response of fiscal variables to expenditure and interest rate shocks depend on the persistency of expenditure as much as on the initial size of debt.

The stability conditions of our model indicate that in the low inflation environment of recent decades, debt stability is not granted. Institutional constraints therefore play a fundamental role to avoid the heavy costs induced by a backward-looking policy aiming to restore debt stability following adverse macroeconomic shocks. These constraints induce a forward-looking approach involving closed-loop policy rules. The rules are important because current levels of debt are mostly unsustainable. Expenditure shocks, induced for example by a recession, or interest cost shocks, induced by a financial crisis, of a comparable size to that observed in the recent past, would almost inevitably induce costly adjustments in tax revenues and living standards. This work highlights that debt sustainability does not depends just on the level of debt. It depends just as much on the level of tax revenues and the persistence of expenditure liabilities.

Appendix I: Steady-state equilibrium values

In steady state, the interest-growth rate differential, $\kappa_t = r_t - \rho_t$, will be constant over time. The level of taxes can then be obtained from the dynamic balance sheet constraint, Equation 2

$$T = E_t + \kappa B. \tag{25}$$

The corresponding steady state level of debt is obtained from Equation 11:

$$\begin{aligned} B - \Big[1 + \kappa + \frac{\alpha + \beta}{d\alpha(1+\kappa)}\Big] B + \frac{1}{d} B = \\ G - \frac{1}{(1+\kappa)d} G + \frac{1}{d\alpha(1+\kappa)} \Big[\gamma(1-d) + r + \gamma\kappa\Big]. \end{aligned} \tag{26}$$

It is easy to verify that, in the limiting case of no discounting and $\kappa = 0$, the steady state value of debt is $B = -\frac{r}{\beta}$, which is *always* negative for positive values of β and positive nominal interest rates.[14] More generally, if we define $a = \frac{1}{d} - \kappa - \frac{\alpha+\beta}{d\alpha(1+\kappa)} = \frac{\alpha(1+\kappa) - d\alpha\kappa(1+\kappa) - \alpha - \beta}{d\alpha(1+\kappa)} = \frac{\alpha\kappa - d\alpha\kappa(1+\kappa) - \beta}{d\alpha(1+\kappa)}$ and $c = -\gamma(1 - d) + r + \gamma\kappa$, the steady-state equilibrium can be expressed as:

$$B = \frac{(1+\kappa)d - 1}{[\alpha\kappa - d\alpha\kappa(1+\kappa) - \beta]a} G + \frac{c}{\alpha\kappa - d\alpha\kappa(1+\kappa) - \beta}. \tag{27}$$

A necessary condition for the denominator of the term on the right to be positive is:

$$\frac{\beta}{\alpha} < \kappa[1 - d(1+\kappa)], \tag{28}$$

When this condition is not satisfied the denominators are negative, and the second term of Equation 26 is always negative. To get positive values of B (net

[14] Negative nominal interest rates could induce a positive steady-state value of debt, but only if policymakers expect them to be a structural, rather than a temporary phenomenon.

debt), the first term of the sum must be positive and large enough to overcome the second. For this $d < \frac{1}{1+\kappa}$ is a necessary condition.

For positive values of κ, and $d < \frac{1}{1+\kappa}$, the policymaker must discount the future very heavily in order to hold debt in steady-state. But in this case stability requires substantial non-linear costs $\frac{\beta}{\alpha}$. Is that realistic in practice?

Table 10 displays an example of such a scenario, assuming steady state values for interest costs of 5 percent and an expenditure share of GDP equal to 0.4375. In this case we get sensible results, but the results change substantially with

Table 10: Steady state values for selected parameters

Parameter values	$\gamma = 0.5$	$\alpha = 0.1$	$\beta = 0.001$	$\kappa = 0.01$	$d = 0.955$	
Steady state values		T=0.444		B=0.63		G=0.4375
Parameter values	$\gamma = 0.5$	$\alpha = 0.5$	$\beta = 0$	$\kappa = -0.0054$	$d = 0.987$	
Steady state values		T=0.431		B=1.13		G=0.4375

variations of the discount rate. For example $d = 0.96$ generates a steady-state value of debt of 0.35, while $d = 0.95$ generates a steady-state debt value of 0.91. However, variations in the equilibrium values for changes in the other variables are less dramatic. In particular, equilibria with positive but reasonable values of debt are compatible with a large range of positive values of κ, α and β values. For example for $\alpha = 0.5$ and $d = 0.95$ steady-state debt B moves to 1.64. But for larger values of α, the results vary a lot with small changes of the discounting factor.

By way of contrast, the condition the condition $d < \frac{1}{1+\kappa}$ is always satisfied for negative values of κ, and steady state levels of debt are positive for a large range of parameters settings: see Table 10. In this type of equilibrium the steady-state value of taxes is lower than that of expenditure; and, as the value of κ grows more negative, the sustainable steady-state level of debt becomes larger. In fact, the equilibrium level of debt may be substantial for even small values of α. With $\kappa = -0.02$, $\alpha = 0.1$, and $\beta = 0.01$, for example, the steady-state value of debt is $B = 0.47$. If the cost of varying taxes is substantial, $\alpha = 0.5$, B remains positive even if there is a high value to issuing debt such as $\beta = 0.2$. In other words, the optimal steady state value of debt may be positive even when the policy-maker is strongly averse to debt, if the cost of varying taxes is large. But when $\beta = 0$, as in the table, the results become sensitive to the value κ. For example, $\kappa = -0.0055$ moves the steady-state level of debt to 2.51; but reducing κ to -0.0053 yields a value of -0.33.

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