ANCHORED INFLATION EXPECTATIONS*

Carlos Carvalho†
Banco Central do Brasil, PUC-Rio

Stefano Eusepi‡
Federal Reserve Bank of New York

Emanuel Moench§
Deutsche Bundesbank

Bruce Preston¶
The University of Melbourne

Abstract
This paper proposes a model of inflation in which changes in long-run inflation beliefs are a state-contingent function of short-run inflation surprises. Expectations are well anchored only when long-run beliefs display small and declining sensitivity to short-run forecast errors. Because of nominal rigidities, shifts in beliefs induce an endogenous inflation trend, in contrast to common specifications of low-frequency movements in inflation, with broad-ranging policy implications. The model, estimated using only US inflation and short-term forecasts from professional surveys, accurately predicts observed measures of long-term inflation expectations for the US and other countries, including several episodes of unanchored expectations.

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†Banco Central do Brasil, PUC-Rio. E-mail: cvianac@econ.puc-rio.br.
‡Federal Reserve Bank of New York. E-mail: stefano.eusepi@ny.frb.org.
§Deutsche Bundesbank. Email: emanuel.moench@bundesbank.de.
¶The University of Melbourne. E-mail: bruce.preston@unimelb.edu.au
Long-run inflation expectations do vary over time. That is, they are not perfectly anchored in real economies; moreover, the extent to which they are anchored can change, depending on economic developments and (most important) the current and past conduct of monetary policy. In this context, I use the term “anchored” to mean relatively insensitive to incoming data. So, for example, if the public experiences a spell of inflation higher than their long-run expectation, but their long-run expectation of inflation changes little as a result, then inflation expectations are well anchored. If, on the other hand, the public reacts to a short period of higher-than-expected inflation by marking up their long-run expectation considerably, then expectations are poorly anchored — Bernanke (2007)

1 Introduction

The notion of anchored inflation expectations provides a central foundation of the theory and practice of monetary policy. Effective management of inflation expectations achieves an efficient short-run trade-off between real economic activity and inflation. Consistent with this principle, for much of the past two decades, long-run inflation expectations have been remarkably stable in many industrialized countries. Yet the stability of inflation expectations is not an inherent feature of an economy, and is surely not independent of economic conditions and the conduct of policy. Indeed one only has to recall the Great Inflation of the 1970s to provide a salutary lesson in the challenges presented by drifting inflation. And more recently, a number of policy makers have expressed concern about the potential consequences of a downward drift in long-term expectations.

Despite such concern, relatively little is known about what actually determines long-run inflation expectations. Standard models used for policy evaluation tend to make stark assumptions: either long-term expectations are consistent with the steady state of the economic model, as assumed in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007); or they are consistent with an exogenously drifting inflation target, as in the models of Smets and Wouters (2003) and Cogley and Sbordone (2008).1 In both cases long-term expectations are pinned down by assumption. There is no behavioral theory of what determines these beliefs. And without such a theory it is difficult to define what it means for inflation expectations to be anchored, and perhaps more importantly, difficult to understand the circumstances under which they might be poorly anchored.

This paper proposes a simple New Keynesian model in which long-term inflation expectations drift as a result of imperfect credibility or, more generally, fundamental uncertainty about the long-run objectives of monetary policy. Integrating quasi-rational learning based on Marcet and Nicolini (2003) into a standard framework of optimal price setting with monopolistic competition and nominal rigidities, the model has the following key features. First, long-term inflation expectations are linked to short-term inflation surprises. This mapping emerges from a filtering problem in which firms attempt to distinguish a potentially time-varying long-term mean of inflation, from large temporary fluctuations, reflecting a variety

1 See also Erceg and Levin (2003), Kozicki and Tinsley (2005), Ireland (2007), Cogley, Primiceri, and Sargent (2010) and Del Negro, Giannoni, and Schorfheide (2015), providing accounts of low-frequency movements during the Great Inflation and subsequent Great Moderation as being due to exogenous shifts in central bank preferences for inflation.
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of transient shocks. Second, the sensitivity of long-term expectations to short-term surprises, defined by the learning gain, is state dependent. In response to large and persistent forecast errors, when agents doubt the constancy of the central bank’s inflation objective, they switch from recursively estimating a time-invariant mean (decreasing gain) to tracking a drifting mean (constant gain). This increases the responsiveness of long-term expectations to short-term inflation developments. In this way, the model gives formal expression to Bernanke’s quote: expectations are anchored when long-run beliefs are relatively insensitive to surprise movements in inflation. Third, because of the existence of nominal rigidities and strategic complementarities in price setting, expectations of future inflation determine current inflation. Expectations are therefore partially self-fulfilling, inducing an endogenous inflation trend that is affected by both fundamental shocks and the prevailing policy regime. Fourth, beliefs exhibit only small deviations from full rationality, satisfying the lower bounds on rationality proposed by Marcet and Nicolini (2003). One key implication is that the expectation formation process is not invariant to changes in policy.

The model is estimated with Bayesian methods using US data on inflation and survey measures of short-term inflation expectations from professional forecasters. Short-term survey forecasts allow directly identifying and testing the link between short-term surprises and long-run beliefs. We then compare long-run inflation expectations implied by the model with various measures available from professional forecasters. This constitutes an external validity test of the model: survey data on long-term inflation expectations are not used in estimation. Furthermore, using the term structure of inflation expectations permits evaluating the information content of these survey data, and, specifically, whether survey participants are responding to macroeconomic developments when reporting long-term expectations, or simply mechanically reporting a central bank’s inflation target.

The estimated model explains both short- and long-term inflation forecasts from professional surveys quite well, providing a better fit when compared to rational expectations or a model with an exogenously determined inflation trend. Given the high degree of self-confirming dynamics in beliefs and the limited role for intrinsic persistence in inflation implied by the estimated parameters, the low-frequency movements in inflation are almost entirely accounted for by firms’ shifting assessments of long-run inflation outcomes. Importantly, the drift in inflation expectations is intimately linked to whether expectations are well anchored or not. The model provides a narrative of US monetary history which accords with conventional wisdom. During the Great Inflation firms systematically under-predict inflation, leading to large and persistent forecast errors and the adoption of a constant-gain estimator. Heightened sensitivity to short-term forecast errors permits better tracking of developments in inflation. However, such beliefs are in and of themselves a contributing source of inflation drift because of self-confirming dynamics. The Volcker disinflation through to the early 1990s under Greenspan continued to see little anchoring of expectations, with large persistent negative forecast errors resulting in a downward adjustment of long-term inflation expectations. After this time, firms accumulate evidence suggesting a time-invariant inflation target, leading to better anchored expectations. Indeed, during the financial crises and recovery,

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3These ideas are closely connected to the literature on predictor selection. See Brock and Hommes (1997) and Branch (2004) for important contributions to this agenda.
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Inflation expectations have moved relatively little — a dividend of this outcome. Because the model provides a fit of the data comparable to standard accounts with an exogenous trend, higher inflation targets during the 1970s are not required to explain inflation data. The broad sweep of these insights are borne out in US household data, from the Michigan Survey, and also professional forecasters in a range of OECD countries. Furthermore, the model provides tight predictions for the evolution of long-term inflation expectations in these countries, even using only the posterior distribution of parameters obtained for the US to generate predictions. Episodes of un-anchored or poorly anchored inflation expectations are detected in several countries, especially over the past ten years.

The empirical results adduce evidence supportive of a large literature on bounded rationality and learning, concerned with modeling long-run beliefs as a function of short-term forecast errors — see Sargent (1993), Evans and Honkapohja (2001), Evans and Honkapohja (2009) and Eusepi and Preston (2016) — and, for this reason, share much in common with the class of beliefs studied here. These alternative expectation formation mechanisms have important policy implications that are, in certain circumstances, quite different from the standard rational expectations framework — see Eusepi and Preston (2016) for a recent survey. In the simple model of this analysis, it is shown that an aggressive monetary policy toward inflation weakens the feedback effect of beliefs on realized inflation by appropriately constraining aggregate demand. In addition, since the expectations formation process is not invariant to the policy regime, more aggressive policies induce learning mechanisms with lower the sensitivity of long-term beliefs to surprises. In absence of strong feedback from beliefs to aggregate dynamics, it is no longer optimal for individual agents to adjust their beliefs as strongly.

Additionally, the model provides a rationale for time variation in the sensitivity of inflation to measures of economic slack, which form the foundation of the New Keynesian Phillips curve. The apparent insensitivity of inflation to real activity in recent years has engendered much criticism of such models of inflation dynamics. The model predicts that when expectations are anchored, as observed going into and beyond the financial crisis, a given sized innovation in marginal costs should lead to smaller movements in inflation. A corollary of this property is that the propagation and amplification of economic disturbances is fundamentally different to a rational expectations analysis of the model. And while developed in partial equilibrium, the ideas translate directly to a general equilibrium environment, in which all structural shocks and multiple policy regimes would shape the evolution of long-run inflation expectations.

The specification of a state-contingent learning gain renders the state-space representation of the model non-linear, requiring the use of techniques that are novel in the context of the macroeconomics literature. In particular, the model is estimated using Bayesian methods and the marginalized particle filter of Schönp, Gustafsson, and Nordlund (2005). This approach permits exploiting the conditionally linear structure of the model to deliver a numerically efficient and accurate estimation algorithm. The model provides an estimate of the entire distribution of possible gains characterizing the sensitivity of a firm’s long-run beliefs to short-run forecast errors at any point in time. While the mean estimate of the learning gain identifies the average sensitivity of beliefs, and, therefore, directly measures the degree to which expectations might be thought to be anchored, the distribution of this key state variable also conveys information about the risks to anchored inflation expectations.
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Stability in the estimated mean gain need not imply stability in the estimated variance of the gain. The latter is therefore a natural metric of central bank credibility in periods in which reported long-term expectations exhibit relatively little variability. The approach provides a richer interpretation of what constitutes anchored expectations than approaches based on pass-through regressions of either macroeconomic news or movements in short-term expectations to long-term expectations as developed by Lamla and Draeger (2013), Buono and Formai (2016), Gurkaynak, Levin, and Swanson (2010), Beechey, Johannsen, and Levin (2011) and Strohsal and Winkelmann (2015). More generally, because inference is based solely on a model of inflation and inflation expectations, the posterior distribution avoids conflating movements in expectations with those in term and risk premia which might be less relevant for wage and price inflation.

2 A Model with Endogenous Inflation Drift

In this section we introduce a simple model of inflation and inflation expectations with three key features. First, short- and long-term inflation expectations are linked by a signal extraction problem. Second, the sensitivity of long-term expectations to short-term surprises is state dependent. Third, because of the existence of nominal rigidities and strategic complementarities in price setting, expectations feed back into current inflation.

Theory of Price Setting. A continuum of monopolistically competitive firms face a Rotemberg (1982) price-setting problem. Given subjective beliefs $\hat{E}_t$, each firm $f \in [0,1]$ maximizes the expected present discounted value of profits

\[ \hat{E}_t = \sum_{T=t}^{\infty} \Lambda_{t,T} \Gamma_T (f) \]

by choice of $P_t (f)$ subject to the demand and profit functions

\[ Y_t (f) = \left( \frac{P_t (f)}{P_t} \right)^{-\theta_t} Y_t \]

\[ \Gamma_t (f) = Y_t (f) \left( \frac{P_t (f)}{P_t} - S_t \right) - \Phi \left( \frac{P_t (f)}{P_{t-1} (f)} - e^{\pi_{t}^{p}} \right)^2 \]

for all $T \geq t$, where $P_t$ and $Y_t$ give the aggregate level of prices and output in period $t$, and $S_t$ a real marginal cost function. The exogenous time-varying elasticity of demand across differentiated goods satisfies $\theta_t > 1$, with mean $\bar{\theta}$. The quadratic costs of price adjustment are determined by price movements relative to an inflation index, which in logs is given by

\[ \bar{\pi}_{t}^{p} = \gamma \pi_{t-1} \]

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In this way the paper makes contact with various literatures concerned with central bank communications policy and credibility. See, for example, Orphanides and Williams (2005), Kozicki and Tinsley (2005) and Eusepi and Preston (2010) for applications of adaptive learning models to these issues; Hommes and Lustenhouwer (2015) for a model of central bank credibility with heterogeneous expectations; and Bomfim and Rudebusch (2000) and Gibbs and Kulish (2015) for explorations of the role of anchored expectations in the costs of disinflation.

See Natoli and Sigalotti (2017) and Ciccarelli, Garcia, and Montes-Galdon (2017) for recent reduced-form contributions using financial market data to proxy long-term inflation expectations. See also Jain (2013) for a novel use of the Survey of Professional Forecast data to infer how well expectations are anchored.
a linear function of the previous-period’s aggregate inflation rate, \( \pi_{t-1} = \ln \left( \frac{P_{t-1}}{P_{t-2}} \right) \).

The constant \( \Phi > 1 \) scales the size of adjustment costs. The parameters \( 0 \leq \gamma \leq 1 \) measures the degree of price indexation. When setting prices in period \( t \), firms value future streams of income according to the stochastic discount factor \( \Lambda_{t,T} \) which in the non-stochastic steady state takes the value \( \beta^{T-t} \), for \( 0 < \beta < 1 \) and all \( T > t \).

Deriving the first-order condition and taking a log-linear approximation around a zero inflation steady state provides the optimal price

\[
p_t^* (f) = \alpha p_{t-1}^* (f) - \alpha (\pi_t - \pi_t^p) + \alpha \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \left( \xi_p s_T + \beta (1 - \alpha) (\pi_{T+1} - \pi_{T+1}^p) \right) + \alpha \hat{\theta}_t \mu_t
\]

where

\[
p_t^* (f) = \ln \left( \frac{P_t (f)}{P_t} \right) ; \quad \hat{\theta}_t = \ln \left( \frac{\theta_t}{\hat{\theta}_t} \right) ; \quad \tilde{\mu}_t = - \frac{\hat{\theta}_t}{\theta - 1}
\]

and \( \xi_p = \alpha^{-1} (1 - \alpha) (1 - \alpha \beta) \) with \( 0 < \alpha < 1 \), the model’s stable eigenvalue.\(^8\) The markup shocks \( \tilde{\mu}_t \) are normally distributed and serially independent process. Optimal price setting requires firms to project the future evolution of marginal costs and aggregate inflation, adjusted for indexation.\(^9\) Aggregating over the continuum in a symmetric equilibrium, where firms set identical prices, \( P_t (f) = P_t \), and hold the same subjective beliefs, \( \hat{E}_t = \hat{E}_t \), provides the aggregate supply curve

\[
\pi_t - \gamma \pi_{t-1} = \xi_p \tilde{\mu}_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \left[ \xi_p s_T + (1 - \alpha) \beta (\pi_{T+1} - \gamma \pi_T) \right]. \tag{1}
\]

### Aggregate demand and monetary policy

The model is closed with a theory of marginal costs, monetary policy and specification of beliefs. To keep matters simple, follow Evans and Honkapohja (2003) and model the evolution of the output gap (the level of output in deviations from its natural level under flexible prices) according to the dynamic aggregate demand curve

\[
x_t = \hat{E}_t x_{t+1} - \left( \iota_t - \hat{E}_t \pi_{t+1} - \hat{\varphi}_t \right) \tag{2}
\]

which is an implication of households’s first-order conditions for optimality and where \( \hat{\varphi}_t \) measures demand shocks, a co-variance stationary exogenous process with \( 0 < \rho < 1 \).\(^10\)

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\(^6\)To the first order this is equivalent to a model of Calvo price setting in which firms, when not optimally re-setting prices, adjust prices according to the index \( \pi_t^p \).

\(^7\)As we only present a first-order approximation to a partial equilibrium model of price setting, other details of the stochastic discount factor are irrelevant.

\(^8\)In a model of Calvo price setting \( \alpha \) corresponds to the probability a firm cannot adjust their price in any given period. While it is well understood the predictions of Rotemberg and Calvo pricing are different at non-zero average rates of inflation, Cogley and Sbordone (2008) show in a closely related model with Calvo pricing it is the assumption of an exogenous inflation trend, rather than the point of approximation, that is most relevant to explaining US inflation data.

\(^9\)See Preston (2005) and Eusepi and Preston (2016) for discussions of optimal price setting under arbitrary beliefs.

\(^10\)This is not the optimal consumption decision for the canonical New Keynesian model under arbitrary beliefs, though suffices given the focus on price setting and inflation expectations — see Preston (2005) for a discussion.
Monetary policy is determined by the rule
\[ i_t = \hat{E}_t \pi_{t+1} + \phi_\pi \left( \hat{E}_t \pi_{t+1} - \gamma \pi_t \right) + \hat{E}_t x_{t+1} + \phi_\varphi \tilde{\varphi}_t + \phi_\mu \tilde{\mu}_t \]  
(3)

where the policy parameters satisfy \( \phi_\pi > 0 \), with \( \phi_\varphi \) and \( \phi_\mu \) unrestricted, specifying that interest rates respond to both inflation and output-gap expectations, and the structural shocks.\(^{11}\) Combining (2) and (3) to eliminate the interest rate, and assuming for expositional simplicity that marginal costs are equal to the output gap (the standard New Keynesian model predicts they are proportional quantities), yields the following simple equation for the marginal cost
\[ s_t = -\phi_\pi \hat{E}_t (\pi_{t+1} - \gamma \pi_t) + (1 - \phi_\varphi) \tilde{\varphi}_t - \phi_\mu \tilde{\mu}_t. \]  
(4)

where
\[ \tilde{\varphi}_t = \rho \tilde{\varphi}_{t-1} + \tilde{\varepsilon}_t. \]

**Rational Expectations Equilibrium.** Under rational expectations agents know the central bank’s inflation target, so that \( \bar{\pi}_t = 0 \) for all \( t \). Subsequent text refers to the object \( \bar{\pi}_t \) interchangeably as the inflation target; the long-run inflation mean; or the inflation drift. Combining (1) and (4) and applying mathematical expectations gives the equilibrium evolution of inflation
\[ \pi_t = \gamma \pi_{t-1} + \rho \varphi_{t-1} + \eta_t \]  
(5)

where
\[ \varphi_t = \frac{\xi_p (1 - \phi_\varphi)}{1 - (\beta - \phi_\pi \xi_p) \rho} \tilde{\varphi}_t; \quad \varepsilon_t = \frac{\xi_p (1 - \phi_\varphi)}{1 - (\beta - \phi_\pi \xi_p) \rho} \tilde{\varepsilon}_t; \quad \mu_t = (\xi_p - \phi_\mu) \tilde{\mu}_t, \]
and \( \eta_t = \varepsilon_t + \mu_t \).

**Imperfect Knowledge.** To solve for the optimal price, firms must forecast future inflation and marginal costs in (1). We assume agents do not know the long-run mean of inflation and must estimate it from available data. They do this using an econometric filter to be described, and the statistical model
\[ \pi_t = \gamma \pi_{t-1} + (1 - \gamma) \bar{\pi}_t + \rho \varphi_{t-1} + e_t \]  
(6)

where \( e_t \) denotes the forecast error, and \( \bar{\pi}_t \) is to be estimated. This measurement equation assumes firms have perfect information about the short-run dynamics of the economy in (5). They understand marginal costs evolve according to relation (4), and that transitional dynamics of inflation are governed by the process \( \varphi_t \) and lagged inflation through the degree of price indexation reflected in \( \gamma \). Beliefs about trend inflation satisfy the property
\[ \hat{E}_{t-1}^f \bar{\pi}_T = \bar{\pi}_t \]
for all \( T \geq t \), which Kozicki and Tinsley (2001) call a shifting end-point model. Because firms do not account for future revisions in their estimates of the inflation target when making period \( t \) decisions, the solution is an example of the anticipated-utility approach — see Kreps (1998), Sargent (1999) and Eusepi and Preston (2016).

\(^{11}\) The choice of rule is not arbitrary. Specific choices of policy parameters delivers a rule that implements optimal discretion for the standard New Keynesian model and welfare theoretic loss function. See Evans and Honkapohja (2003) for a derivation.
Evaluating expectations in the aggregate supply curve, (1), provides the true data-generating process
\[ \pi_t = \gamma \pi_{t-1} + (1 - \gamma) \Gamma \pi_t + \rho \varphi_{t-1} + \eta_t \] (7)
where
\[ \Gamma = \frac{1 - \alpha}{\alpha} \left[ \frac{\alpha \beta}{1 - \alpha \beta} - \phi_\pi \right] \]
Subjective and model-consistent predictions differ only insofar as \( \Gamma \) differs from unity. The coefficient \( \Gamma \) measures the degree of feedback from beliefs to actual inflation and depends on the stance of monetary policy, indexed by \( \phi_\pi \).\footnote{Eusepi and Preston (2016) for an extensive discussion on the role of policy design in mitigating instability from self-referential dynamics.} For policies responding weakly to inflation, the composite parameter \( \Gamma \) takes values near unity under standard assumptions, so the model has the property that beliefs are nearly self-confirming — the forecasting model is nearly consistent with the true data-generating process.\footnote{That is for \( \beta \) close to unity.} In the case of a more aggressive monetary policy, determined so that \( \Gamma \) takes values near zero, beliefs do not affect inflation dynamics. While later empirical work and counterfactual analysis gives emphasis to \( \Gamma \), the dependency of this composite parameter on monetary policy should be kept in mind as a central determinant of its value. Variations in the degree of self-referentiality can be interpreted as variations in the stance of monetary policy.

**Learning and the anchoring of expectations.** It remains to specify the process by which firms update their estimate of the average long-run inflation rate. Following Marcet and Sargent (1989) and Evans and Honkapohja (2001), we assume beliefs are revised according to the learning algorithm linking the current estimate of the inflation mean to the last prediction error\footnote{A complicated simultaneity is resolved by assuming the estimate, \( \pi_t \), depends on the previous-period’s forecast error. This is standard in the learning literature.}
\[ \bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \times f_{t-1} \]
where
\[ f_t = \pi_t - \hat{E}_{t-1} \pi_t = (1 - \gamma) (\Gamma - 1) \pi_t + \eta_t. \]
Similar in spirit to Marcet and Nicolini (2003), the gain is determined by
\[ k_{t+1} = \begin{cases} k_t + 1, & \text{if } \left| \hat{E}_{t-1} \pi_t - \mathbb{E}_{t-1} \pi_t \right| \leq \nu \sigma_\eta \\ \bar{g}^{-1}, & \text{otherwise} \end{cases} \] (8)
where \( \bar{g}, \nu > 0 \) are parameters; \( \hat{E}_{t-1} \pi_t \) is the firm’s one-period-ahead forecast; \( \mathbb{E}_{t-1} \pi_t \) the model-consistent one-period-ahead expectations; and \( \sigma_\eta = \sigma^\epsilon + \sigma^\mu \) denotes the standard deviation of the innovation \( \eta_t \). At any point in time, the gain is described by one of two possible regimes. The first corresponds to a decreasing-gain algorithm: the estimate \( \bar{\pi}_t \) is the recursive least-squares estimate of the inflation mean. Alternatively, the gain is constant, and the estimate corresponds to a weighted average of past inflation, where older observations are given geometrically decreasing weight; the higher the value of the gain \( \bar{g} \), the less weight
is given to old observations. This learning algorithm can be interpreted in terms of the Kalman filter. The model of the inflation drift is

$$\bar{\pi}_t = \bar{\pi}_{t-1} + w_t$$

where \( w_t \) has mean zero and variance which can take two values corresponding to different regimes: \( \sigma_w^2 = \{ \bar{\sigma}_w^2, 0 \} \). In the first regime, the drift is governed by a random walk. The constant-gain algorithm can be interpreted as the steady-state Kalman updating of this model. The second regime corresponds to a constant mean for inflation, giving a decreasing-gain algorithm.\(^{15}\)

The state-dependent gain \( k_t \) safeguards against structural change. A constant-gain estimator, \( \bar{g} \), produces better forecasts when the economic environment changes. Using this estimator beliefs do not converge to the central bank’s inflation target, even in a stationary environment. In contrast, a decreasing-gain estimator, such as ordinary least squares, provides better forecasting performance in an economy where the mean of inflation is time invariant. Furthermore, it can deliver asymptotic convergence to the true constant mean.

The final component of the model specifies the choice of forecasting model at any point in time. Our approach is motivated by parsimony and the requirement to have a criterion based on past forecast errors. These objectives are met by assuming shifts in the learning gain regime are determined by

$$\left| \hat{E}_{t-1} \pi_t - E_{t-1} \pi_t \right| \leq \nu \sigma^y$$

(9)

where the parameter \( \nu \) regulates how alert firms are to model misspecification. The switch to a constant gain \( \bar{g} \) occurs when the subjective prediction is sufficiently far from the model-consistent forecast, where the distance is normalized by the volatility of the innovations to the inflation process. One might express surprise that firms base their forecasting model on a criterion involving model-consistent expectations — which they are assumed not to know. The assumption is made in the as if tradition in economics and represents a reduced-form description of a richer framework of model specification tests, in which statistical tools are deployed to detect time variation in the intercept of the forecasting model. The important assumption is that the criterion to switch gains depends on past forecasting errors.

To see this dependency and obtain further intuition on the properties of the algorithm, use the forecast error in (9) to write

$$\left| \hat{E}_{t-1} \pi_t - E_{t-1} \pi_t \right| = \left| (1 - \gamma) (\Gamma - 1) \bar{\pi}_t \right|$$

$$= \left| (1 - \gamma) (\Gamma - 1) \left[ \pi_0 + \sum_{\tau=0}^{t} k^{-1}_\tau f_\tau \right] \right|$$

(10)

given some initial conditions \( \bar{\pi}_0 \), \( f_0 \) and \( k_0 \). The distance (10) tends to be large when forecast errors happen to be of the same sign for several periods. This pushes \( \bar{\pi}_t \) away from its long-run mean, and drives a wedge between the perceived and true model dynamics

\(^{15}\)See Bullard (1992) for a discussion. Note, however, that the updating rule, (8), is not optimal because it does not fully account for the regime switching in the variance of \( \bar{w}_t \).
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of inflation. Finally, the left hand side of the criterion (9) has the further advantage of not introducing additional free parameters. Other criteria could easily be adopted, such as a simple parametric function of forecast errors in the recent past. However, they would require estimating these additional parameters and are therefore eschewed on the grounds of parsimony.  

Regime uncertainty and rationality. The learning mechanism described above allows agents to track regime changes. Agents do not know a priori all possible regimes they might face, along with the transition probabilities among such regimes, including whether they are time invariant. Over the sample period being considered it is simply implausible to assume agents should have known ex ante the entire set of policy regimes, let alone their relative likelihood at any point in time. An implication is the learning mechanism considered is not the outcome of an optimal filter from a statistical perspective. Nevertheless, following Marcet and Nicolini (2003) beliefs satisfy a lower bound on rationality, and are optimal in a sense made precise in section 5.

While agents are learning about the regime in place and fear possible regime changes, we assume that the model environment is in fact time invariant. In particular the monetary authority implements a time-invariant rule to promote price stability — an average of zero inflation. The assumption of a time-invariant policy regime is a simplification that gives emphasis to the role of beliefs in aggregate inflation dynamics, and follows a long tradition in macroeconomics — see Smets and Wouters (2007), for example. We later establish that beliefs about the long-run inflation rate ultimately converge to the central bank’s inflation target. Formally, under certain parameter restrictions, the estimate, \( \bar{\pi}_t \), converges to zero with probability one. In other words, beliefs converge to the rational expectations equilibrium in (5). Integrating actual regime change in this setup is left for future research.

Anchored inflation expectations. This belief structure captures precisely Bernanke’s (2007) notion of anchored expectations. In a stable macroeconomic environment, where agents do not make large and persistent forecast errors, the distance between subjective firm forecasts and objective model-consistent forecasts tends to be small. As a result, because of the adoption of a decreasing-gain algorithm, beliefs exhibit relatively low (and declining) sensitivity to forecasting errors, and inflation expectations are anchored. Contrariwise, in periods of relative turmoil, where agents are systematically surprised, long-term inflation expectations become highly sensitive to forecast errors. In such times, beliefs are poorly anchored, providing our model-based definition of unanchored inflation expectations.

Whether long-term expectations are anchored or not is intimately connected to the endogenously determined low-frequency movements in inflation. Equations (7) and (10) show that as \( \bar{\pi}_t \) drifts from its rational expectations equilibrium value of zero, such movements are

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16See for example Milani (2014). Note using the actual forecast error under subjective beliefs would deliver a criterion that depends only on that contemporaneous error — not the recent history.

17Evans, Honkapohja, and Williams (2010) shows that filters of the kind studied here represent an approximation to the Bayesian filter and can also be interpreted as the maximally robust estimator emerging from a robust control decision problem.

18Eusepi and Preston (2010) propose a related notion of anchored expectations in a model of learning. In that paper expectations were said to be anchored when beliefs are consistent with the central bank’s policy rule. The analysis here broadens this notion to give focus to long-term inflation expectations and their determinants.
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partially self-confirming, as they subsequently determine inflation and forecast errors. Large forecast errors sustain the constant-gain regime and, therefore, the volatility in the inflation drift. In contrast, in a decreasing-gain regime, forecast errors become less important for long-term expectations over time, as the gain converges to zero. As a result, \( \bar{\pi}_t \) fluctuates less, producing smaller forecast errors and reinforcing the choice of a decreasing gain.

**Model Summary.** The evolution of the gain in (8) is given by the transition equation

\[
k_{t+1} = f_k(k_t, \bar{\pi}_t) \quad (11)
\]

\[
= I(\bar{\pi}_t) \times (k_t + 1) + (1 - I(\bar{\pi}_t)) \times \bar{g}^{-1}
\]

where

\[
I(\bar{\pi}) = \begin{cases}
1, & \text{if } |(1 - \gamma)(\Gamma - 1)\bar{\pi}| \leq \nu \sigma \eta \\
0, & \text{otherwise}.
\end{cases}
\]

Given the specification of the gain, \( k_t \), and a sequence of exogenous processes \( \{\varphi_t, \eta_t\} \), model dynamics are described by the pair of equations

\[
\pi_t = \gamma \pi_{t-1} + (1 - \gamma) \Gamma \bar{\pi}_t + \rho \varphi_{t-1} + \eta_t \quad (12)
\]

\[
\bar{\pi}_{t+1} = [1 + f_k^{-1}(k_t, \bar{\pi}_t)(1 - \gamma)(\Gamma - 1)] \bar{\pi}_t + f_k^{-1}(k_t, \bar{\pi}_t) \eta_t. \quad (13)
\]

Two points are worth underscoring. First, the estimated mean inflation rate is a first-order auto-regressive process with time-varying persistence and sensitivity to innovations. In contrast to models of the kind proposed by Cogley and Sbordone (2008), in which low-frequency movements in inflation are captured by an exogenous process, the evolution of \( \bar{\pi}_t \) depends on the structural innovations of the model. The model endogenously generates low-frequency properties of inflation through the interaction of the firm’s inference problem and optimal price setting.\(^{19}\)

3 Estimation

The data and observation equation. We estimate the model with Bayesian methods using US data on both inflation and survey measures of short-term inflation expectations from professional forecasters. The estimation strategy employs only these data for inference on model parameters. The use of short-term forecasts directly identifies the forecast errors that are central to the mechanism proposed here. In fact, conditional on observing short-term forecasts, the updating algorithm (8) implies fairly tight predictions for the evolution of the estimated inflation mean. Having tied our hands in this fashion, model success is evaluated by a comparison of model-implied predictions of long-term inflation expectations with equivalent measures available from survey data.

\(^{19}\)Kozicki and Tinsley (2005) and Ireland (2007) also study models in which structural shocks affect the central bank’s inflation target in an exogenously determined way. Our analysis departs from these papers by permitting an endogenous dependence of low-frequency movements in inflation on structural shocks. Models of exogenous drift are discussed further in Section 7.
For data comparability in a number of estimation exercises, we use the log-difference in the CPI as the measure of inflation. Four survey measures of CPI inflation forecasts are used: one- and two-quarter-ahead forecasts from the Survey of Professional Forecasters (SPF); and two measures of six-month-ahead forecasts from the Livingston Survey. The first Livingston Survey measure is computed as the growth rate between the forecast of the CPI level six-months ahead and the last monthly price level available to forecasters at the time of the survey. The second measure is the growth rate between the forecast of the CPI level six-months ahead and the forecast of the current CPI level. The latter is a more accurate measure of inflation expectations, though is only available for a shorter sample.

The sample spans 1955Q1-2015Q4. The survey data are available over different sample sizes and at different frequencies. The SPF measures are available starting in 1981Q3, at a quarterly frequency. The Livingston survey is available only at a bi-annual frequency, but its first measure of six-months-ahead forecasts is available since the beginning of the sample, while the second starts only in 1992Q2. Reflecting the structure of these data, the model observation equation is

\[
\begin{bmatrix}
\pi_t \\
E^\text{SPF}_{t+1} \\
E^\text{SPF}_{t+2} \\
E^\text{LIV}_1 \left( \frac{1}{2} \sum_{i=1}^2 \pi_{t+i} \right) \\
E^\text{LIV}_2 \left( \frac{1}{2} \sum_{i=1}^2 \pi_{t+i} \right)
\end{bmatrix} = \pi^* + H_t \begin{bmatrix} \bar{\pi}_t \\ \xi_t \end{bmatrix} + R_t \omega_t
\]

where \(\xi_t = (\eta_t, \varphi_t, \pi_t)\); \(\pi^*\) is the mean inflation rate; and \(\omega_t\) measurement error attached to both the survey data and CPI inflation. The measurement error on inflation captures the fact the CPI measure of inflation exhibits substantial quarter-to-quarter volatility that is not incorporated in short-term forecasts and is understood to be temporary. For example, the technical appendix shows that while CPI inflation is substantially more volatile than the GDP deflator, the survey-based forecasts of these two variables are very similar. Finally, the matrices \(H_t\) and \(R_t\) are time varying because of missing observations.

**Marginalized particle filter.** Because the model is non-linear, standard inference methods based on the Kalman filter cannot be employed. However, the model has a conditionally linear structure which can be exploited to deliver a relatively efficient estimation of this non-linear model. Partition the model states into a subset of non-linear variables \((\bar{\pi}_t, k_t)'\) and a subset of linear variables, \(\xi_t\), and write the state-space representation as

\[
k_t = f_k (\bar{\pi}_{t-1}, k_{t-1})
\]

\[
\bar{\pi}_t = f_{\bar{\pi}} (\bar{\pi}_{t-1}, k_{t-1}) + A_{\bar{\pi}} (\bar{\pi}_{t-1}, k_{t-1}) \xi_{t-1}
\]

\[
\xi_t = f_\xi (\bar{\pi}_{t-1}, k_{t-1}) + A_\xi (\bar{\pi}_{t-1}, k_{t-1}) \xi_{t-1} + S_\xi \epsilon_t
\]

where \(\epsilon_t = (\epsilon_t, \mu_t)\) are normally distributed innovations, and remaining coefficients are defined in the appendix. The model can then be estimated using the Marginalized Particle Filter proposed by Schön, Gustafsson, and Nordlund (2005): using Bayes rule, the linear state variables \(\xi_t\) are marginalized out and estimated using the linear Kalman filter. The nonlinear
state variables are estimated using the particle filter — see, for example, Kitagawa (1996). This improves both the accuracy of the estimation process and the speed of computation. The details of the algorithm are in the appendix.

**Estimated parameters.** Table 1 shows prior and posterior distributions for each parameter. The posterior distribution is obtained by first computing the mode of the distribution. In a second step we use the Metropolis-Hastings algorithm to compute the full distribution. Convergence is evaluated using the Gelman and Rubin potential-scale-reduction factor, which was well below 1.01 for all estimated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior Mean</th>
<th>Prior St. Dev.</th>
<th>Prior Mode</th>
<th>Posterior Mean</th>
<th>5 percent</th>
<th>95 percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^*$</td>
<td>Normal</td>
<td>2.250</td>
<td>0.400</td>
<td>2.461</td>
<td>2.472</td>
<td>2.102</td>
<td>2.880</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Gamma</td>
<td>0.050</td>
<td>0.040</td>
<td>0.020</td>
<td>0.029</td>
<td>0.012</td>
<td>0.052</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>Gamma</td>
<td>0.100</td>
<td>0.090</td>
<td>0.125</td>
<td>0.145</td>
<td>0.103</td>
<td>0.203</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.265</td>
<td>0.115</td>
<td>0.128</td>
<td>0.090</td>
<td>0.173</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.265</td>
<td>0.926</td>
<td>0.891</td>
<td>0.813</td>
<td>0.948</td>
</tr>
<tr>
<td>$\rho_\phi$</td>
<td>Beta</td>
<td>0.500</td>
<td>0.200</td>
<td>0.874</td>
<td>0.877</td>
<td>0.832</td>
<td>0.917</td>
</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>Inv.-Gamma</td>
<td>0.100</td>
<td>2.000</td>
<td>0.086</td>
<td>0.084</td>
<td>0.071</td>
<td>0.098</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>Inv.-Gamma</td>
<td>0.100</td>
<td>1.000</td>
<td>0.371</td>
<td>0.359</td>
<td>0.301</td>
<td>0.416</td>
</tr>
<tr>
<td>$\sigma_{o,1}$</td>
<td>Inv.-Gamma</td>
<td>0.100</td>
<td>1.000</td>
<td>0.264</td>
<td>0.277</td>
<td>0.217</td>
<td>0.335</td>
</tr>
<tr>
<td>$\sigma_{o,2}$</td>
<td>Inv.-Gamma</td>
<td>0.100</td>
<td>1.000</td>
<td>0.043</td>
<td>0.042</td>
<td>0.035</td>
<td>0.050</td>
</tr>
<tr>
<td>$\sigma_{o,3}$</td>
<td>Inv.-Gamma</td>
<td>0.100</td>
<td>1.000</td>
<td>0.020</td>
<td>0.021</td>
<td>0.014</td>
<td>0.028</td>
</tr>
<tr>
<td>$\sigma_{o,4}$</td>
<td>Inv.-Gamma</td>
<td>0.100</td>
<td>1.000</td>
<td>0.071</td>
<td>0.073</td>
<td>0.063</td>
<td>0.084</td>
</tr>
<tr>
<td>$\sigma_{o,5}$</td>
<td>Inv.-Gamma</td>
<td>0.100</td>
<td>1.000</td>
<td>0.048</td>
<td>0.049</td>
<td>0.041</td>
<td>0.059</td>
</tr>
</tbody>
</table>

*Note:* The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table 1: **Prior and Posterior Distribution of Structural Parameters**

The data permit fairly tight identification of key model parameters. Measurement errors on the survey forecasts have small variance. As discussed further below, this provides confidence that the core mechanism of the model is reasonably well identified. The measurement error on inflation implies price changes in the model are somewhat less volatile than CPI inflation, but track its movements fairly closely. The sample average inflation rate, $\pi^*$, has a posterior mean of about 2.5%, in annualized terms, and the 90% posterior interval ranges from 2.1% to 2.9%. The parameter $\gamma$ is tightly distributed around 0.1, suggesting a small degree of price indexation. This is consistent with the observation of Cogley and

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21 As it was not possible to obtain a reliable Hessian using our particle filter algorithm, we started the Metropolis-Hastings step using a diagonal matrix with prior variances on the diagonal. The transition probability function was iteratively updated using short chains of between 20,000 and 40,000 draws. This process is repeated until the resulting variance-covariance matrix is stable. The variance-covariance matrix so obtained is used to generate 5 samples of 200,000 draws. A step size of 0.2 gave a rejection rate of 0.64 in each sample.

22 In particular, model-implied inflation remains significantly more volatile than the GDP deflator. See the additional Appendix for details.
Sbordone (2008) that once low-frequency movements in inflation are properly accounted for, there is little evidence in favor of price indexation. Concomitantly, the feedback effects from drifting beliefs, as measured by $\Gamma$, are substantial, with the 90% interval between 0.8 and 0.95. Later counterfactual analysis demonstrates self-confirming beliefs are the driving force behind the rise and fall of inflation over the sample period, and central to whether long-term expectations are well anchored or not.

Turning to the parameters defining the learning algorithm, $\nu$ has a posterior mean of 0.03 with 90% credible interval spanning 0.01 and 0.05. To evaluate these magnitudes, for each draw from the posterior distribution, use (10) to measure the minimum absolute distance between $\bar{\pi}_t$ and its true value of zero required for firms to switch to a constant gain. Using the 90% posterior interval, this threshold ranges between 0.3% and 0.8% in annual terms. More concretely, the mean threshold implies that as $\bar{\pi}_t$ drifts outside the interval $2% < \pi^* + \bar{\pi}_t < 3%$ firms switch to the constant-gain regime. This reveals an important property of the belief structure: the estimated threshold predicts firms are responsive to small deviations from the true inflation mean. However, as shown in the next section, it takes a sequence of persistent and potentially large shocks to shift the estimated inflation drift significantly, especially under decreasing-gain learning.

The constant gain has mean posterior estimate 0.14. Values of the gain within the 90% interval imply firms give minimal weight to observations older than five years. For example, at the boundaries of the 90% posterior interval, corresponding to $g = 0.1$ and 0.2, the weight on observations two-years old is 0.2 and 0.4 respectively, with five-year weights being negligible. The posterior distribution for $g$ therefore suggests that when inflation expectations are unanchored, long-term inflation expectations are quite sensitive to short-term forecast errors.

4 Model Predictions

Figure 1 displays the model’s fit of short-term inflation forecasts. In each panel, the dashed grey line denotes CPI inflation; the solid black line displays the median prediction, while the grey shaded area shows the 95% credible set. Consistent with the small size of the observation errors in Table 1, the model-implied short-term forecasts correspond closely with the survey forecasts represented in red and blue: forecast surprises are well disciplined by the data. This permits testing the state-dependent link between short-term surprises and long-term expectations, which is the defining mechanism of the model.

To this end, we compare model-based predictions with available survey-based long-term forecasts. Because many surveys do not cover the entire sample, we use a number of measures from various surveys to build a comprehensive picture of long-term inflation expectations. Prior to the late 1970s there are no professional forecast data. We therefore use the five-to-ten-year-ahead forecasts from the Michigan Survey, restricted to the period 1974Q2-1977Q2. After this time the following professional forecast data are available. For one-to-ten-year-ahead average inflation forecasts: the Decision Makers Poll Survey (1978Q3-1980Q4); Livingston Survey (1990Q2-2015Q4); Blue Chip Economic Forecasts (1979Q4-1980Q4).

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23 As mentioned above the model-implied inflation, while somewhat less volatile, tracks closely observed CPI inflation.

24 We study the behavior of the Michigan survey in section 8.
Anchored Inflation Expectations

One-quarter ahead (SPF)

Two-quarters ahead (SPF)

Six-months ahead (Livingston)

Figure 1: Short-Term Forecasts.

The top and middle panels show the evolution of one- and two-quarters-ahead forecasts from SPF; The bottom panel shows two measures of the six-months-ahead forecasts from Livingston. The black lines show median predictions, while the grey area measures the 95% credible interval; the red and blue dots denote survey-based forecasts. Finally, the thin grey line measures CPI inflation.
Anchored Inflation Expectations


Long-term inflation expectations. Figure 2 shows long-term inflation expectations as predicted by the model, along with corresponding survey data. The survey data are given by the red and blue dots in the upper panel, which correspond to the five-to-ten- and one-to-ten-year-ahead forecasts. When multiple surveys are available for the same forecast the figure reports the average forecast across surveys. The solid black line in the top panel measures the median prediction for the five-to-ten-year-ahead expectation, while the grey areas denote the 70% and 95% credible sets; the dashed black line shows the median prediction for the one-to-ten-year-ahead forecast. The proximity of the two model-generated long-term forecasts underscores the low-frequency movement in inflation is for the most part driven by the drift $\pi_t$. The model captures well the evolution of long-term forecasts. The credible sets are fairly tight, especially beyond the 1970s, and the survey-based forecasts lie, for the most part, within the 70% credible set. More broadly, the model captures the mild increase in expectations in the mid-seventies; the surge up to the early 1980s; the gradual decline through to the 1990s; and the stabilization in the 2000s, which persists beyond the crisis period.

Importantly, the results are consistent with contemporary narratives of US monetary history. As is clear from the pattern of short-term forecasts in the middle panel of Figure 1, starting in the early 1970s firms faced persistent positive inflation surprises, leading to poorly anchored inflation expectations, and a switch to a constant gain illustrated in the bottom panel. The constant-gain regime lasts until the mid-1990s, because of persistent negative surprises, initially during the Volcker disinflation, and, subsequently, the opportunistic disinflation in the early 1990s under Greenspan. Since the late 1990s, conditional on observed inflation and short-term forecasts, the 95% credible set excludes a switch to a constant-gain regime, despite occasionally large forecast errors. Indeed, from Figure 1, these forecast errors are of a similar magnitude to those observed over the 1970s and 1980s. However, the pattern of forecast errors is crucially different: they were not as persistent and, therefore, did not generate large enough deviations in (10) to lead to a drift in inflation beliefs. The model therefore provides a rationale for the observed stability of expectations during this period which include sizable shocks like the 2009 recession.

Endogenous inflation trend and monetary policy. The relative stability of the persistent component of inflation has been the basis of much recent skepticism about models relating inflation to measures of economic slack, such as the Philips curve in its various forms. But the analysis presented here is entirely consistent with the apparent time variation in the sensitivity of inflation to economic activity. In fact the weak link between shocks and inflation dynamics during the financial crisis and beyond is fully consistent with the New Keynesian Phillips curve embedded in this model: it is the by-product of anchored inflation expectations during this period. Conversely, the large and persistent shocks realized over

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25See for example Hall (2011) and Ball and Mazumder (2011).
26It is worth clarifying that our estimation exercise does not allow a clear identification between cost-push and marginal cost shocks, given that we have no observable measures of marginal costs or slack. This is left for further research.
the 1970s and 1980s produced sizable effects on inflation because they led to unanchored expectations. These shocks were therefore transmitted to the persistent component of inflation, creating a strong link between real activity and prices.\textsuperscript{27}

More generally, given the centrality of inflation expectations to modern monetary economics, the endogeneity of the inflation trend has key implications for policy design — the impulse and propagation mechanisms are fundamentally altered. For example, Eusepi, Giannoni, and Preston (2017) show in a simple New Keynesian model that optimal policy under unanchored expectations (large constant gain) exhibits history dependence and resembles optimal commitment under rational expectations. On the other hand, anchored inflation expectations (small constant gain) induce no history dependence, as in the case of discretion under rational expectations.

Prior accounts of the low-frequency movements in inflation, typified by the Great Inflation, rely on purely exogenous specifications of the inflation drift — see, for example, Smets and Wouters (2003), Cogley and Sbordone (2008) and Cogley, Primiceri, and Sargent (2010). While Kozicki and Tinsley (2005) and Ireland (2007) permit more general specifications in which the drift is determined in part by identified disturbances, such as supply shocks, the specification remains exogenous. This matters for interpretation and policy consequences. These papers interpret low-frequency movements as the result of variation in preferences over inflation outcomes. Higher inflation in the 1970s merely reflects the Federal Reserve’s rising tolerance for inflation. In contrast, because the drift is endogenous in our account, the Great Inflation may have arisen despite the Federal Reserve’s commitment to price stability.\textsuperscript{28} In this way, the paper has much in common with Sargent (1999) and Primiceri (2005) which emphasize learning by policy makers rather than price setters.

Finally, it is worth underscoring, the result that long-term expectations have been quite stable over the past decade does not appeal to any informational friction predicting inertia in belief updating. The results suggest that reported survey inflation expectations represent a coherent view of actual inflation developments — not simply mechanical reportage of central bank inflation objectives. Long-term forecasts are adjusted significantly only in response to systematic and sufficiently large forecast errors. Once again: the fact that expectations are anchored in itself affects the size of forecast errors — the stability of expectations arises endogenously in this model. The counterfactuals in Section 6 further clarify this point.

5 Bounds on Irrationality

The central mechanism of the paper concerns the state-contingent mapping of short-run forecast errors to long-run beliefs. The estimation exercise evidences this mechanism is well identified, as the model can account for observed forecast errors implied by survey data without recourse to substantial observation error. Furthermore, model predictions for long-term expectations track closely the empirical counterpart from survey-based measures. While this constitutes credible evidence in support of the theory, it relies on the choice of two free parameters defining the learning algorithm. This section demonstrates these parameters

\textsuperscript{27}Del Negro, Giannoni, and Schorfheide (2015) provides a similar argument in a medium-size DSGE model, where long-term inflation expectations evolve exogenously.

\textsuperscript{28}Eusepi and Preston (2016) provides evidence for a related belief structure, in an empirical medium-scale general equilibrium model of the US economy.
Figure 2: Predictions: Professional Forecasters.

The top panel shows the evolution long-term expectations at five-to-ten-year (5-10Y) and one-to-ten-year (1-10Y) horizons. The black lines show median predictions, while the grey area measures the 70% and 95% credible intervals; the red and blue dots denote survey-based forecasts. The bottom panel shows the evolution of the learning gain.
satisfy a lower bound on rationality — agents cannot hold beliefs that are implausible. As a corollary, it shows that more aggressive monetary policy toward inflation weakens the link between short term inflation surprises and long-term inflation expectations, delivering more strongly anchored beliefs.

**Asymptotic convergence.** A reasonable requirement of any proposed learning algorithm is that firms ought to be able to learn the time-invariant long-run mean of inflation when they operate in a stationary environment. Learning should have the property that the estimate \( \bar{\pi}_t \) converges to zero, the rational expectations equilibrium, with probability one.\(^{29}\) The following formally establishes this result. Suppose firms construct forecasts using a recursive least-squares algorithm. Using standard results in Marcet and Sargent (1989) and Evans and Honkapohja (2001) we have the following Lemma.

**Lemma 1.** Consider the equilibrium evolution of \( \bar{\pi}_t \) under least-squares learning. Provided \( \Gamma < 1 \), then \( P(\bar{\pi}_t \to 0) = 1 \).

**Proof.** Recall from (13)

\[
\bar{\pi}_{t+1} = \bar{\pi}_t + \mathbf{t}^{-1} \times [(1 - \gamma)(\Gamma - 1)\bar{\pi}_t + \eta_t].
\]

The mean dynamics are then approximated by the ordinary differential equation

\[
\dot{\bar{\pi}} = (1 - \gamma)(\Gamma - 1)\bar{\pi}
\]

which is stable, provided \( \Gamma < 1 \). □

This asymptotic result establishes the local stability of the inflation target: so long as initial beliefs are sufficiently close to the true long-run mean of inflation, given enough data firms can learn the objective of the central bank. Now consider the properties of beliefs under a constant-gain algorithm.

**Lemma 2.** Consider the equilibrium evolution of \( \bar{\pi}_t \) under constant-gain learning. Provided

\[
|1 + \bar{g}(1 - \gamma)(\Gamma - 1)| < 1
\]

then \( \bar{\pi}_t \) evolves according to an auto-regressive process of order one, with mean 0.

**Proof.** From (13), the dynamics of the inflation drift satisfy

\[
\bar{\pi}_{t+1} = [1 + \bar{g}(1 - \gamma)(\Gamma - 1)]\bar{\pi}_t + \bar{g}\eta_t
\]

which is a statistical process with the desired properties if the condition of the proposition is met. □

Beliefs about trend inflation are ergodically distributed around the true long-run mean of inflation. Combining these two results gives the convergence result.

\[^{29}\text{Formally, we consider a log-linear approximation in the neighborhood of price stability (zero inflation).}\]
Proposition 1. Consider the equilibrium evolution of $\pi_t$ under the learning algorithm (8).
Provided
\[ |1 + \bar{g} (1 - \gamma) (\Gamma - 1)| < 1 \]
then $P(\pi_t \to 0) = 1$.

Proof. From Lemma 1 there exists $0 < \bar{\nu} < \nu$ such that if $\pi_t \in S^\nu$, where $S^\nu \equiv (\pi_t : |\pi_t| < \bar{\nu} \sigma_t)$, then $P(\pi_t \to 0) > 0$. The probability is not one, because for small values of $t$ sufficiently large shocks can shift $\pi_t$ outside of $S^\nu$, which defines the domain of attraction for local stability. However, from Lemma 2, in the constant gain regime $\pi_t$ must visit the stable set $S^\nu$ infinitely often. Therefore, $P(\pi_t \to 0) = 1$: eventually the estimator will not escape the domain of attraction of the ordinary differential equation, guaranteeing convergence.

Given sufficient data, firms will always learn the long-run mean of inflation.\textsuperscript{30} This property distinguishes our model from various other contributions in the literature which seek to study questions of central bank credibility, and, specifically, whether inflation expectations are anchored or not. For example, Orphanides and Williams (2005) and Kozicki and Tinsley (2005) develop models of central bank credibility in which agents must estimate the inflation target using a constant-gain algorithm. While imperfect knowledge has implications for monetary policy design, both models have the property that agents cannot ever learn the time-invariant inflation target. More recently, papers such as Hommes and Lustenhouwer (2015) explore similar questions in a model of heterogeneous beliefs and predictor selection. While the composition of predictors is endogenous to the environment, the model nonetheless has the property that the ergodic distribution of beliefs never converges to the central bank’s inflation target.

Optimality. The second lower bound on rationality is given by a criterion proposed by Marcet and Nicolini (2003). We show that within the adopted class of learning algorithm beliefs are in a certain sense optimal. Specifically, conditional on a data-generating process where all firms learn using parameters $(\bar{g}, \nu)$, an individual choosing a potentially different parameter pair $(\bar{g}', \nu')$ should not produce much better forecasts than the parameters $(\bar{g}, \nu)$ according to a mean-squared error criterion. Formally, the choice of beliefs must satisfy
\[ \bar{E} [f_t(\bar{g}, \nu)]^2 \leq \min_{\bar{g}', \nu'} \bar{E} [f'_t(\bar{g}, \nu, \bar{g}', \nu')]^2 + \epsilon \]
for some small $\epsilon > 0$. We approximate the above moments using the mean-squared error from a sample of size $T$, averaged over $S$ replications, where each replication represents a simulation of the model at the modal estimates. Consistent with the sample size in estimation, take $T = 264$, and $S = 200,000$. This is done for all candidate beliefs on a grid covering the interval $\bar{g} \in [0.001, 0.3]$ and $\nu \in [0.001, 0.1]$.

\textsuperscript{30} Much of the literature on learning and monetary policy can be interpreted as models of central bank credibility, and as answering the question of whether expectations are well anchored or not. As one example, Eusepi and Preston (2010) provide theoretical results on central bank communication, showing certain types of information ensure consistency of beliefs with monetary policy strategy — providing a well-defined notion of anchored expectations — which improves stabilization outcomes of a given monetary policy framework. However, such theoretical analyses, relying on asymptotic convergence results, do not provide an account of dynamics when beliefs are poorly anchored.
Figure 3: **Optimal choice of $\nu$ and $\bar{g}$**

These panels show the ratio between the mean-squared error from predictions of a measure-zero agent using alternative values of $\nu$ and $\bar{g}$ and that from the representative agent using modal estimates. The true data-generating process is obtained assuming agents use the modal estimates. The top panel assumes the feedback parameter $\Gamma$ at the mode; the bottom panel assumes $\Gamma = 0.4$ or lower feedback effects.
Figure 3 shows the ratio of the mean-squared errors of the counterfactual beliefs relative to the baseline model, as a function of \((\bar{g}', \nu')\) in two different economies. The top panel provides the results for the estimated model. For the grid considered, the estimated parameters \((\bar{g}, \nu)\) are optimal, with other forecasting models exhibiting a deterioration in performance. There is no incentive to deviate from these beliefs. This is explained by the high degree of self-referentiality in the system. Because movements in beliefs are in large part reflected in the data, it is optimal to maintain those beliefs. Indeed, the bottom panel shows the same exercise in a counterfactual economy exhibiting substantially less feedback than at the mode: \(\Gamma = 0\). As discussed in section 2, a more aggressive policy towards inflation lowers \(\Gamma\). As a result, an individual firm would find it desirable to adopt a lower value of the gain and a higher switching threshold. This shows how the expectation formation mechanism is not independent of policy. A more aggressive policy toward inflation endogenously lowers the sensitivity of long-term inflation expectations to short-term surprises and, with that, limits the drift observed in inflation.

6 Counterfactuals

The two defining features of the model are that beliefs and optimal pricing decisions combine to generate self-referential dynamics, and the learning gain is state dependent. The following counterfactuals isolate the contributions of each of these properties. All counterfactuals are generated by simulating the economy using the posterior distribution of the model parameters, and the smoothed distribution of initial states and structural innovations.

6.1 Understanding the Endogenous Trend

The role of self-referential beliefs. The model captures the low-frequency movement in inflation, as well as properties of long-term forecast data. A novel feature is that the model determines the drift endogenously, as an outcome of self-referential feedback: through optimal price setting, shifts in beliefs affect inflation which, in turn, shapes beliefs. This interplay can produce substantial variation in beliefs for certain sequences of forecast errors. Importantly, low-frequency drift in beliefs is directly related to episodes of poorly anchored inflation expectations.

To illustrate this connection, we run the following counterfactual simulation. Assume \(\Gamma = 0\) to eliminate the feedback from beliefs to actual inflation. Consistent with the discussion in Section 2, this case is equivalent to a monetary policy which systematically adjusts to fully offset shifts in beliefs. Furthermore, assume firms update their estimates of \(\bar{\pi}_t\) using a constant-gain algorithm in all periods, so that \(\nu = 0\). Long-run beliefs are therefore quite sensitive to inflation surprises, even though we have shown in the previous section that this would not have been optimal. The top panel in Figure 4 shows the median prediction for the agents’ long-term expectations (solid black line) together with the 95% credible interval in this counterfactual economy. Also shown are the long-term survey forecasts, together with the 95% credible set from the predicted long-term inflation expectations of the baseline model (light green). Even with the assumed high responsiveness to inflation surprises, inflation

\[31\] To better visualize the result, the Figure shows a smaller set of parameter values relative to the size of the grid used in the simulation. The result, however, holds for all parameters in the grid.
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expectations in the counterfactual economy exhibit limited drift. Absent feedback effects, inflation expectations fluctuate very little. This illustrates how in this model the sensitivity of inflation to disturbances can vary over time, as it depends jointly on the updating of beliefs and policy regime. Moreover, such property is consistent with an otherwise standard New Keynesian Phillips curve.\textsuperscript{32}

**The role of timing of switching beliefs.** How important is the timing of switches in the forecasting algorithm? Can beliefs that are relatively insensitive to new information be counterproductive from the perspective of a central bank trying to engineer a disinflation? Long-term inflation expectations plateau briefly at around 4% in the late 1980s, before continuing to decline over the period of the ‘opportunistic disinflation’. Suppose at this time agents become convinced the observed decline in inflation represents the end of the Federal Reserve’s commitment to disinflation. Believing inflation to be close to its time-invariant mean, firms switch to a decreasing-gain algorithm to construct forecasts. The middle panel in Figure 4 shows this counterfactual. In this scenario, long-term inflation expectations decline very slowly and remain above 3% at the end of 2015. Again, self-referential dynamics play a central role in explaining slow convergence. Because beliefs display less sensitivity to new information, negative inflation surprises lead to smaller downward revisions in long-term inflation expectations, which results in actual inflation declining more gradually. This hinders the ability of firms to learn the true underlying inflation mean.

To gauge further the importance of these self referential effects we run an additional experiment. Assume now inflation expectations for all firms are updated using a constant gain, so that inflation evolves exactly as in the baseline model. Now suppose a measure-zero firm updates their beliefs using a decreasing-gain algorithm. In contrast to the previous simulation, the measure-zero agent faces a data-generating process which has inflation declining much faster. Moreover, their own expectations do not feed back into actual inflation outcomes. The dashed black line demonstrates the evolution of this firm’s inflation expectations. Despite using a decreasing-gain algorithm, when there is no self-referentiality inflation expectations converge noticeably faster, falling below 3% by 2005 as compared to 2015.

**Stable expectations: “well anchored” or “good luck”?** From the early 2000s long-term inflation expectations are very stable. Through the lens of the model, this stability reflects neither inattention nor inertial adjustment on the part of professional forecasters. Similarly, it does not reflect good luck as captured by favorable disturbances. Indeed, large forecast errors arising from substantial energy and commodity price changes, and the events of the financial crisis underscore anchored expectations are not simply the result of a stable underlying inflation process. Rather, the stability of long-term expectations is due to the relatively small, and decreasing, learning gain, a dividend of well-anchored inflation expectations. What if firms remained skeptical of the Federal Reserve’s commitment to price stability, and never switched to a decreasing gain in the mid-to-late 1990s? With the US economy experiencing the same shocks as in the baseline model, the bottom panel of Figure 4 demonstrates inflation under constant-gain learning would have exhibited substantial volatility. In contrast to the baseline prediction, long-term inflation expectations would have

\textsuperscript{32}Drift in inflation is not simply the mechanical outcome of a specific choice of learning algorithm. Rather, beliefs about future inflation, combined with optimal price setting and, therefore, the actual inflation process, jointly determine the drift.
Figure 4: **Counterfactuals: Endogenous Inflation Trend**

The three panels show the predicted behavior of long-term expectations under alternative counterfactual simulations described in the main text. The black line denotes median predictions; the grey area shows the 95% credible interval in the counterfactual; the green lines show the 95% credible interval under the baseline model; the blue and red dots measure long-term forecasts from surveys.
been very uncertain, with the credible interval ranging from 0.5% to 4%. In addition, toward the end of the sample, the model’s predictions imply some downside risk to inflation expectations, accompanied by median predictions trending below 2%.

6.2 Alternative Models

**Time-varying gain.** A key feature of the proposed learning algorithm is the discontinuous nature of the changes in the learning gain. If firms doubt the forecasting performance of their model, they switch to a constant-gain algorithm, potentially leading to a substantial change in the degree of sensitivity of long-run beliefs to new information. Such adjustments in the gain afford considerable flexibility to adapt to shifting economic circumstances. To evaluate the role of the discontinuity, we now consider the adaptive-step-size algorithm proposed by Kushner and Yin (2003). This belief structure permits the learning gain to be adjusted continuously, leading to a more gradual adjustment in the sensitivity of long-run beliefs to short-term surprises. This algorithm was first used in macroeconomics by Kostyshyna (2012) in the context of the hyperinflation model of Marcet and Nicolini (2003).

The algorithm can be expressed as

\[ g_t = G(g_t-1 + \nu f_{t-1}V_{t-1}) \]

\[ V_t = (1 - g_t-1)V_{t-1} + f_{t-1}, \quad V_0 = 0 \]

where \( g_t \) denotes the time-varying gain; \( V_t \) measures discounted past forecast errors, \( f_t \); and the function \( G(\cdot) \) satisfying

\[ G(g_t-1 + \nu f_{t-1}V_{t-1}) = \begin{cases} 
  g_{t-1} + \nu f_{t-1}V_{t-1}, & \text{if } g_- < g_{t-1} + \nu f_{t-1}V_{t-1} < g^+ \\
  g^+, & \text{if } g^+ < g_{t-1} + \nu f_{t-1}V_{t-1} \\
  g^-, & \text{if } g^- > g_{t-1} + \nu f_{t-1}V_{t-1}.
\end{cases} \]

The parameter \( \nu \) now captures how the learning gain is adjusted in response to past forecast errors. The bounds \( g^+ \) and \( g^- \) ensure that the algorithm is well behaved.\(^{33}\) When the current forecast error, \( f_t \), has the same sign as discounted past errors, \( V_t \), the gain increases. This has similar intuition to our baseline algorithm: the gain changes in response to persistent forecast errors of the same sign.

Figure 5 shows outcomes of a counterfactual simulation under this belief structure.\(^{34}\) The parameter \( \nu \) is chosen to replicate the rise in inflation expectations during the early 1970s, and is set equal to 0.003. We also consider the alternative value of 0.001 for robustness. The top panel demonstrates expectations fail to track the survey data, and are substantially more volatile in the latter part of the sample. The bottom panel reveals the source of difficulty is that the gain coefficient rises in the 1970s, but fails to decline much at all after 1980. This is

\(^{33}\)In the numerical analysis that follows they are set to \( g^+ = 0.6 \) and \( g^- = 0.01 \) and the estimate for \( \bar{g} \) never reaches these boundaries.

\(^{34}\)Similarly to the above analysis, the counterfactuals use the posterior parameter distributions for the inflation process and the filtered shocks obtained from our baseline estimation.
Figure 5: Counterfactuals: Alternative Models.

The three panels show the predicted behavior of long-term expectations under alternative counterfactual simulations described in the main text. The black line denotes median predictions; the grey area shows the 95% credible interval in the counterfactual; the green lines show the 95% credible interval under the baseline model; the blue and red dots measure long-term forecasts from surveys.

true for both values of $\nu$ plotted, though there is a level difference in the degree of sensitivity to past forecast errors. This counterfactual adds evidence that our threshold algorithm is better able to capture the relation between long-term expectations and short-term surprises, relative to algorithms which continuously update the gain.

7 Model Comparison

The predictive density in Figure 2 shows the baseline model tracks very well the evolution of long-term inflation expectations. Here we perform a more formal assessment of the model by comparing its predictive likelihood against three alternatives: the rational expectations version of our baseline model where $\tilde{\pi}_t^* = 0$ in every period; a model where beliefs are updated
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using a constant-gain algorithm — equivalent to setting \( \nu = 0 \) in our baseline model; and a model where \( \bar{\pi}_t^* \) evolves exogenously according to the auto-regressive process

\[
\bar{\pi}_{t+1} = \rho \bar{\pi}_t + \varepsilon_t^\pi
\]

where the innovation is independent of the structural shocks in \( \eta_t \). In contrast to earlier models, the inflation trend is treated as an exogenous process. As a statistical description of inflation, this final class of models has been studied in a range of contexts, ranging from reduced-form, partial equilibrium and general equilibrium analyses — see, for example, Smets and Wouters (2003), Stock and Watson (2007), Cogley and Sbordone (2008), Cogley, Primiceri, and Sargent (2010), Del Negro, Giannoni, and Schorfheide (2015), Mertens (2016) and Cecchetti, Feroli, Hooper, Kashyap, and Schoenholtz (2017). What they have in common is the inflation trend is highly persistent, with auto-regressive coefficient in the neighborhood of, or equal to, unity. In the subsequent exercise we set \( \rho = 0.99.35 \).

Because the process for the inflation trend is exogenous, we set \( \Gamma = 1 \) in (13).

Following Del Negro and Eusepi (2011), we evaluate how these models, estimated to fit inflation and short-term survey forecasts, perform in predicting long-term inflation expectations. To achieve this we compute the predictive likelihood

\[
p(y^{LT}|y^{ST}, M_j) = \int p(y^{LT}|y^{ST}, \theta, M_j) p(\theta|y^{ST}, M_j) d\theta = p(y^{LT}, y^{ST}|M_j) / p(y^{ST}|M_j)
\]

where \( y^{LT} \) includes the two series measuring long-term inflation expectations shown in Figure 2: the five-to-ten- and one-to-ten-year inflation forecasts. The vector \( y^{ST} \) includes CPI inflation and all the short-term survey forecasts used in the estimation of the baseline model. Finally \( \theta \) contains the estimated parameters in each model. This vector includes two additional measurement errors associated with the long-term forecasts.\(^{36}\) The predictive likelihood measures the fit of long-term inflation expectations conditional on the parameter distribution delivering the best fit of both short-term inflation expectations and inflation. In other words, we evaluate each model’s fit of long-term expectations by using as a prior the posterior parameter distribution obtained using only short-term forecasts and inflation. As shown above, it can be computed by taking the ratio of the marginal likelihoods resulting from estimating the model using two different datasets: one including \( (y^{ST}) \), and the second additionally including long-term forecasts, \( (y^{ST}, y^{LT}) \).

Table 2 compares the marginal and predictive likelihood for the baseline model and the three alternative models. The baseline model performs best, providing additional evidence for the proposed learning mechanism. Figure 6 gives intuition for the superior fit of our baseline model. The three panels show the predictive density for each alternative model (estimated using only short-term forecasts), compared to the survey-based long-term expectations and the median predictions from the baseline model. The model under rational expectations, which delivers the worst predictive likelihood, fails to deliver the large increase

\(^{35}\)This choice is consistent with a range of papers in the literature. It could easily be estimated without consequence.

\(^{36}\)We set the standard deviation of both measurement error innovations to 0.15.
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\[
\ln p(Y_{1,T}^{ST}) \quad \ln p(Y_{1,T}^{ST}, Y_{1,T}^{LT}) \quad \ln p(Y_{1,T}^{LT} | Y_{1,T}^{ST}) \\
\text{Dataset without LT Expectations} \quad \text{Dataset with LT Expectations} \quad (2) - (1)
\]

<table>
<thead>
<tr>
<th>Model</th>
<th>(1)</th>
<th>(2)</th>
<th>(2) - (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>-567.68</td>
<td>-677.97</td>
<td>-110.29</td>
</tr>
<tr>
<td>Rational Expectations</td>
<td>-588.51</td>
<td>-727.34</td>
<td>-138.83</td>
</tr>
<tr>
<td>Constant Gain</td>
<td>-571.43</td>
<td>-686.66</td>
<td>-115.23</td>
</tr>
<tr>
<td>Exogenous $\pi^*_t$</td>
<td>-579.44</td>
<td>-698.55</td>
<td>-119.11</td>
</tr>
</tbody>
</table>

*Note:* The table shows the log-marginal likelihood for the four alternative models. The first column corresponds to the case where long-term expectations are not used in the estimation, while the second column shows the marginal likelihood when long-term expectations are added to the data set. Finally, the third column shows the predictive likelihood for each model.

**Table 2: Model Comparison**

In long-term expectations during the 1970s and 1980s, while predicting excessively volatile expectations in the 2000s. The constant-gain model performs well until the mid-2000s and then produces a run-up in inflation expectations in the mid-2000s, when inflation had been volatile. Finally, the model with exogenous inflation mean under-predicts long-term inflation expectations in both the 1980s and 1990s, and produces excessively volatile expectations in the 2000s. The estimated volatility of the innovation to the inflation drift is the result of a trade-off between fitting the large variations in the inflation trend during the first part of the sample with the stability in the latter part. As shown by Mertens (2016), for example, even higher-dimensional models with an exogenous inflation trend which use several measures of inflation and economic activity in the estimation present similar shortcomings in fitting long-term inflation forecasts.

**8 Predicting other Long-term Forecasts**

This final section demonstrates the basic belief structure provides a good characterization of other survey data of inflation expectations. One concern about the results is the good fit might simply reflect we are showing in-sample predictions. After all, the parameter distribution is chosen to match the joint behavior of US inflation and short-term professional forecasts. For this reason, we document evidence supporting the proposed theory of belief formation for different types of economic agents (households rather than professional forecasters), and for a range of different countries.
Figure 6: Model Comparison

The three panels show three alternative models estimated on the same observables as our baseline model.
8.1 Household Expectations

The Michigan Survey has collected data on short-term inflation forecasts since nearly the beginning of our sample, and on long-term forecasts since 1975. Despite the long sample, the survey presents some challenges. First, in contrast with professional surveys, short-term forecasts are measured as one-year-ahead forecasts, so we can measure quarterly inflation surprises only indirectly. Second, survey participants are not asked to forecast a specific price index. In what follows we therefore continue to use CPI as our measure of inflation, acknowledging this is only a noisy measure of the underlying inflation rate about which households form expectations.\(^{37}\) Third, short-term inflation forecasts display a substantial upward bias in the last fifteen years when compared to CPI inflation. To avoid modeling this bias directly, we mitigate the problem by using median forecasts, instead of the mean. Furthermore, following the finding of Coibion and Gorodnichenko (2012) that the difference between professional forecasters and Michigan forecasts is related to oil prices, we purge the bias by regressing the difference between short-term professional and household forecasts on oil prices. Our measure of household expectations is then given by the sum of the survey of professional forecasts data and the residual from the regression.\(^{38}\)

With these limitations in mind, the observation equation for household expectations is

\[
E_t^{\text{Mich}} \left( \frac{1}{4} \sum_{i=1}^{4} \pi_t + i \right) = \pi^* + H_t^\prime \xi_t + R_t o_t.
\]

The observation error on inflation now additionally captures the discrepancy between the CPI and the survey participants’ notion of the inflation measure being predicted. We then use the posterior distribution from the baseline model using professional forecasts to infer an estimate of household beliefs about trend inflation. That is, conditioning on CPI inflation and the Michigan Survey short-term inflation expectations, we predict household long-term expectations. We assume the measurement error on inflation and the short-term forecast correspond to those estimated for inflation and the one-period-ahead professional forecast in the benchmark estimation. These assumptions are inconsequential given the small size of estimated measurement errors.

Figure 7 shows the model predictions for the five-to-ten-year-ahead expectations, along with the corresponding Michigan survey forecasts. The pattern matches broadly that for professional forecasters, adducing complementary evidence to Malmendier and Nagel (2016) that adaptive learning structures provide a reasonable description of household forecasts.\(^{39}\) However, there are differences. First, the middle panel shows the evolution of one-year-ahead forecasts for both households (red dots) and professional forecasters (blue dots).\(^{40}\) Interestingly, household forecasts decline more sharply than professional forecasts from the

\(^{37}\)Note that measurement error on CPI inflation can at least partially address this data limitation.

\(^{38}\)The transformed and un-transformed measures are shown in the technical appendix.

\(^{39}\)Malmendier and Nagel (2016) exploit the cross-section providing evidence that different age-cohorts use different constant gains to form inferences about future inflation. The results are also consistent with Lamla and Draeger (2013) which documents a decline in correlation between changes in households’ short-term and long-term expectations over a similar sample in the Michigan Survey micro data.

\(^{40}\)These are computed using the median response from the Livingston survey. A similar pattern obtains if the SPF are used, but the sample is shorter.
The panels show model predictions for long-term forecasts (top), short-term forecasts (middle) and the learning gain (bottom). The black solid (dashed) line denotes the median prediction; the grey area measures the 70% and 95% credible intervals; in the top panel, the red (green) dots indicate long-term survey forecasts from the Households Michigan survey (Professional forecasters); Finally the green solid line in the top panel shows the mean prediction from the baseline model.

Figure 7: Predictions: Households Forecasts.
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early 1980s to the early 1990s. As a result, the model on average predicts a lower path for long-term inflation expectations. In fact, over the same period long-term household forecasts from the Michigan survey (red dots) are consistently below professional forecasts (green crosses). In line with the pattern of short-term forecasts, household and professional long-term expectations move in sync during the 1990s. Second, while the median prediction is that expectations remained anchored in the aftermath of the financial crises, model predictions are more uncertain than for professionals: the model attributes a substantial probability to long-term expectations not being fully anchored during that period. This is consistent with the temporary increase in household long-term forecasts during the crisis.

More generally, the model predictions depart from observed expectations in two dimensions. First, the model falls short of capturing the sharp increase in expectations in the late 1970s, suggesting other factors beyond observed inflation surprises triggered such a large spike. Second, while the predicted long-term forecasts mimic reasonably closely the variation in long-term survey forecasts, by the end of the sample the model predicts long-term expectations to be somewhat below realized forecasts on average. This discrepancy can be traced to the role of oil prices in affecting Michigan Survey forecasts since the late 1990s. Indeed, the model can be used to gauge to what extent changes in energy prices have shifted long-term inflation expectations. Recall the short-term inflation forecasts have been corrected for the influence of oil price movements, but not the long-term forecasts. Conditional on the model being right, the difference between model predictions and the corresponding survey data therefore measures the effects of oil on long-term survey forecasts. We can then conclude that the effects of oil prices on average long-term forecasts over the 2000s are not particularly large.

8.2 Professional forecasts: other countries

The sample of international data comprises the following countries: France, Germany, Italy, Spain, Japan, Canada, Switzerland and Sweden. Inflation expectations are measured using data available from Consensus Economics. While the Consensus Economics data set includes short-term and long-term professional survey forecasts for a wide set of countries, it presents two challenges for estimation. First, forecasts in this data set are made on a year-over-year basis. As for the Michigan Survey, this formulation prevents a clean identification of the mechanism of the model, which links one-step-ahead forecast errors to the beliefs about long-term inflation, \( \pi_t \). Second, in contrast with the US forecast data, for most countries expectations data are only available from 1991, providing a limited time series for estimation. The following discussion details how each of these complications is handled.

Mapping model concepts to the data. Estimation employs available Consensus Economics inflation forecasts for the one- and two-years-ahead horizons. In contrast with SPF forecasts which have a constant horizon, independently of when they are polled, in the Consensus Economics data set the forecast horizon is shrinking in each quarter, and the target forecast becomes progressively less uncertain towards the end of the year. Since forecast horizons are different in each quarter, we have only one observation for each particular forecast per year.

To map the data concept into the model concept, year-over-year forecasts can be approximated as a weighted average of quarterly forecasts at different horizons, with tent-shaped
weights. For the estimation we use six sets of forecasts. The first two are forecasts for the current year, made in the first and second quarter of the year. The remaining four are forecasts for the next year made in each quarter of the current year. Common to all these forecasts is most of the weight is given to quarterly forecasts ranging from one-to-four-quarters ahead. Details on the observation equation can be found the in Appendix. Similarly to the Michigan survey, these are reasonably interpreted as short-term forecasts. Figure 8 plots short-term forecasts with an horizon below one year and above one year respectively, along with the CPI for each country. These forecasts are fairly close and respond to current inflation developments.

**Confronting a short sample.** To handle the short available sample for the international data (survey data are available only from 1991), which prohibits sharp identification of the inflation trend using only CPI data (available from the mid-to-late 1950s), we employ posterior information from the US model to shape the priors adopted in estimation for these countries. In particular, the US posterior is used as a prior for all parameters except for the steady-state inflation rate and all observation errors. For these parameters, we use the same prior distributions as specified for the US. One final assumption is made. When simulating the posterior distribution of foreign parameters, the foreign likelihood function is scaled by the parameter $\lambda^F$ so that

$$P^F (\theta^F | Y_t^F, Y_t^{US}, \theta^{US}) = L(Y_t^* | \theta^{US}, \theta^F)^{\lambda^F}L(Y_t^{US} | \theta^{US})p(\theta^{US})p(\theta^F).$$

Smaller values of the parameter $\lambda^F$ imply model predictions are more closely tied to the US posterior distribution. The presented results consider the case $\lambda^F = 0.05$. This weight delivers a posterior distribution of the common parameters that is extremely close to the distribution for the US model, while gaining some information about country-specific mean inflation rates and observation errors. This provides a clear test of the out-of-sample forecasting power of the model estimated on US data.

**Model predictions.** Figure 9 presents model predictions for long-term inflation expectations. For each country the top panel provides the model-implied long-term inflation forecast and 95% credible interval (the solid black line and grey band), along with the corresponding survey data (red dots). The dashed line provides the CPI inflation rate. The bottom panel displays the estimated gain, along with 50%, 70% and 95% credible intervals.

We offer the following observations. First, the model characterizes the evolution of long-term forecasts surprisingly well, despite model parameters being largely inferred from US data. For most of the sample, the survey-based forecasts are inside the 95% credible set. Second, while the precise timing differs, in general long-term expectations are more stable from the early-to-mid 1990s, with the estimated median gain declining for all countries relative to the 1980s. Perhaps not surprisingly, long-term inflation expectations in Canada follow similar dynamics to the US: the model places high probability on anchored expectations in recent years. However, several countries exhibit episodes in which expectations are poorly anchored. For example, Germany and Spain experience a temporary rise in the estimated gain after German reunification and during the Global Financial Crisis, respectively.

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42 Details can be found in the appendix.
43 The appendix documents the evolution of short-term expectations used for the estimation and prediction, together with the results for two additional countries: Netherlands and Norway.
These panels show the evolution of CPI inflation and short-term survey forecasts. The green line and the blue dots denote short-term survey forecasts of different horizons, from Consensus Economics.
Also, Italy, Germany and France display a high degree of uncertainty about the size of the gain during the crisis period, despite the median estimate being consistent with anchored expectations. At the end of the sample, all of these countries show some downside risk to long-term inflation expectations according to the 95% credible set.

Japan displays more prolonged periods of heightened sensitivity of long-term inflation expectations to short-term forecast errors, especially in the latter part of the sample. This is reflected in both the median estimate and also the 95% credible set which provides a metric of the risks that long-term expectations are not well anchored. Despite several years of deflation, both predicted and actual long-term inflation expectations remain above zero. Moreover, they appear to revert to about 2% over the latter part of the sample. Similarly to the US, this suggests the observed behavior of long-run expectations does not simply reflect inertia in survey responses by forecasters; rather, it is consistent with the observed pattern of surprises. Finally, Switzerland provides a second example of an economy for which our model implies unanchored expectations. However, here the model predictions and observed forecast data are in stark contrast, as survey-based long-term forecasts have remained fairly stable despite persistent forecast errors.

This serves to introduce our final observation. The comparison between the model-predicted paths and survey-based forecasts suggests that in some countries short-term forecast errors might not have been the only determinant of long-term expectations. For example, for much of the sample Japan had higher reported long-term inflation expectations than predicted by the model. This possibly reflects various fiscal and monetary interventions implemented to combat deflation over the period, or, more recently, concerns about long-term fiscal sustainability. Sweden displays a faster decline in survey-based long-term forecasts compared to model predictions in the early 1990s. This coincides with the announcement and adoption of an inflation targeting regime in 1992 – 1995. And in the case of Switzerland, which displays the largest discrepancy over the final years of the sample, the Swiss National Bank operated a formal exchange rate policy to limit appreciation of the Swiss Franc. This might have granted the policy framework an independent source of credibility despite short-term inflation behavior.

In each of these cases, announcement effects and other policy-related factors might have plausibly shifted expectations beyond what is justified by short-term inflation forecast errors alone. And in practice, it will almost certainly be the case that firms and other market participants will condition long-term inflation forecasts on a range of information, not least other indicators of the state of the macro-economy, such as short-term interest rates and output. What is remarkable about our results, for both the US and other countries, is that measures of short-term inflation expectations go a long way in explaining the dynamics of long-term inflation expectations. They don’t explain everything, but they clearly have strong predictive content for longer-term developments.
Figure 9: **Selected OECD countries**

These panels show model predictions for long-term inflation forecasts (top) and the learning gain (bottom). Black solid line denotes the median; the grey areas measure the 50th, 70th and 95th credible intervals; the red dots denote the five-to-ten inflation forecasts from Consensus Economics.

**EMU Countries**

France

Germany

Italy

Spain

**Other Countries**

Canada

Japan

Sweden

Switzerland
9 Conclusions

This paper proposes a model of long-term inflation expectations with three defining characteristics. First, long-term inflation expectations are linked to short-term surprise movements in inflation through a signal extraction problem. Second, the sensitivity of long-term expectations to short-term surprises is state-contingent, exhibiting periods of heightened and lessened response. Third, because of nominal rigidities, firms beliefs about future inflation determine current inflation. Together these assumptions generate a testable theory, in which long-term inflation expectations are time varying, and determined endogenously within a structural theoretical framework. As such the theory departs from many recent analyses which treat trend inflation as an exogenous process, argued to represent time variation in a central bank’s preference over long-run inflation outcomes. Perhaps most importantly, the model provides a meaningful definition of anchored expectations, as times when long-run beliefs exhibit relatively little sensitivity to short-run surprise movements in inflation.

Using only contemporaneous inflation and short-run survey forecast data in estimation, the model provides an impressive account of survey data on long-term inflation expectations. Indeed, model comparison exercises reveal the endogenous trend model provides a better fit to the exogenous trend model, and various other alternatives. Counterfactual exercises reveal the importance of the state-contingent sensitivity of beliefs and self-referentiality in building a coherent theory of long-term inflation expectations. Importantly, the model can be used to evaluate the consequences of poorly anchored inflation expectations. Should expectations become poorly anchored in the United States they could vary widely relative to recent history, with obvious complications for short-run stabilization policy.
REFERENCES


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A Appendix (Not for Publication)

Here we present the marginalized particle filter and smoother. Recall the model is summarized by the following equations

\[
\pi_t = (1 - \gamma) \Gamma \bar{\pi}_t + \gamma \bar{\pi}_{t-1} + \varphi_t + \mu_t
\]

\[
\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \times f_{t-1}
\]

\[
k_t = I(\bar{\pi}_{t-1}) \times (k_{t-1} + 1) + (1 - I(\bar{\pi}_{t-1})) \times g^{-1}
\]

\[
f_t = (1 - \gamma) (\Gamma - 1) \bar{\pi}_t + \mu_t + \epsilon_t
\]

\[
\varphi_t = \rho \varphi_{t-1} + \epsilon_t,
\]

where the function \( I(\bar{\pi}) \) is described as

\[
I(\bar{\pi}) = \begin{cases} 
1, & \text{if } |(1 - \gamma)(\Gamma - 1) \bar{\pi}| \leq \nu \sigma^\eta \\
0, & \text{otherwise.}
\end{cases}
\]

The model can then be re-written as

\[
k_t = f_k(\bar{\pi}_{t-1}, k_{t-1})
\]

\[
\bar{\pi}_t = f_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}) + f_k(\bar{\pi}_{t-1}, k_{t-1})^{-1} \times \eta_{t-1}
\]

\[
\eta_t = \mu_t + \epsilon_t
\]

\[
\varphi_t = \rho \varphi_{t-1} + \epsilon_t
\]

\[
\pi_t = (1 - \gamma_p) \Gamma f_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}) + (1 - \gamma_p) \Gamma f_k(\bar{\pi}_{t-1}, k_{t-1})^{-1} \eta_{t-1} + \gamma \bar{\pi}_{t-1} + \rho \varphi_{t-1} + \epsilon_t + \mu_t.
\]

where

\[
f_k(\bar{\pi}_{t-1}, k_{t-1}) = I(\bar{\pi}_{t-1}) \times (k_{t-1} + 1) + (1 - I(\bar{\pi}_{t-1})) \times g^{-1},
\]

\[
f_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}) = \left[ 1 - (1 - \Gamma)(1 - \gamma) f_k(\bar{\pi}_{t-1}, k_{t-1})^{-1} \right] \bar{\pi}_{t-1}.
\]

We can also re-write the system in matrix notation. One way to write it is by separating linear and nonlinear states. For the linear variables we have:

\[
\xi_t = f_{\xi}(\bar{\pi}_{t-1}, k_{t-1}) + A_{\xi}(\bar{\pi}_{t-1}, k_{t-1}) \xi_{t-1} + S_{\xi} \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix},
\]
where

\[
\xi_t = \begin{bmatrix} \eta_t \\ s_t \\ \pi_t \end{bmatrix};
\]

\[
f_\xi(\bar{\pi}_{t-1}, k_{t-1}) = \begin{bmatrix} 0_{2 \times 1} \\ (1 - \gamma) \Gamma f_\pi(\bar{\pi}_{t-1}, k_{t-1}) \end{bmatrix};
\]

\[
A_\xi(\bar{\pi}_{t-1}, k_{t-1}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho \phi & 0 \\ (1 - \gamma) \Gamma f_k(\bar{\pi}_{t-1}, k_{t-1})^{-1} & \rho \phi & \gamma \end{bmatrix};
\]

\[
S_\xi = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.
\]

For the nonlinear variables we can express

\[
k_t = f_k(\bar{\pi}_{t-1}, k_{t-1})
\]

and

\[
\bar{\pi}_t = f_\pi(\bar{\pi}_{t-1}, k_{t-1}) + A_\pi(\bar{\pi}_{t-1}, k_{t-1}) \xi_{t-1}.
\]

where

\[
A_\pi(\bar{\pi}_{t-1}, k_{t-1}) = \begin{bmatrix} f_k(\bar{\pi}_{t-1}, k_{t-1})^{-1} \\ 0_{2 \times 1} \end{bmatrix}.
\]

Notice that \(k_t\) does not depend on the linear state. In yet another formulation we can express the system in more compact notation:

\[
k_t = f_k(\bar{\pi}_{t-1}, k_{t-1})
\]

\[
\begin{bmatrix} \bar{\pi}_t \\ \xi_t \end{bmatrix} = f(\bar{\pi}_{t-1}, k_{t-1}) + A(\bar{\pi}_{t-1}, k_{t-1}) \xi_{t-1} + \begin{bmatrix} 0 \\ S_\xi \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}
\]

where

\[
f(\bar{\pi}_{t-1}, k_{t-1}) = \begin{bmatrix} f_\pi(\bar{\pi}_{t-1}, k_{t-1}) \\ f_\xi(\bar{\pi}_{t-1}, k_{t-1}) \end{bmatrix}
\]

\[
A(\bar{\pi}_{t-1}, k_{t-1}) = \begin{bmatrix} A_\pi(\bar{\pi}_{t-1}, k_{t-1}) \\ A_\xi(\bar{\pi}_{t-1}, k_{t-1}) \end{bmatrix}
\]

and

\[
\Sigma = E \left( \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix} \right)
\]

is the variance covariance of the innovations.
This notation is used below when computing the smoothed states. Finally, given the data \( Y_T = y_1...y_T \), the model observation equation is

\[
y_t = h_{0,t} + h_{\pi,t}\pi_t + H_t^i\xi_t + R_t^{1/2}e_t^o
\]

where the vectors and matrices \( h_{0,t}, h_{\pi, t}, H_t^i \) and \( R_t \) are defined to be consistent with the timing of available data, and \( e_t \) denotes observation errors.

### A.1 Algorithm for the Marginalized particle filter

This follows Schönh, Gustafsson, and Nordlund (2005). For details of the particle filter we use Kitagawa (1996). We are looking for the following distributions:

\[
p (\xi_t, [\pi_t, k_t] | Y_t) = p (\xi_t | [\pi_t, k_t], Y_t) \times p ([\pi_t, k_t] | Y_t).
\]

The following describes the algorithms. Discussion and proofs are given below.

**Algorithm:**

1. **Initialization.** Choose \( \pi^{(i)}_{1|0}, k^{(i)}_{1|0} \) from some distributions (drawing from normal for \( \pi \) and \( k^{(i)}_{0|0} = \overline{k}_0 \) (or draw from \( \mathcal{U}(0, \overline{g}^{-1}) \)), and \( \xi^{(i)}_{1|0}, P^{(i)}_{1|0} = [\xi_0, P_{1|0}] \), where \( P_{1|0} \) denotes the initial precision matrix in the linear Kalman filter.

2. For each \( t = 1...T \), compute

\[
\Omega_t = H_t^iP_{t|t-1}H_t + R_t
\]

and its inverse. For \( i = 1, ..., N \), evaluate the importance weights

\[
q_t^{(i)} = p \left( y_t | \pi^{(i)}_{t|t-1}, \xi^{(i)}_{t|t-1} \right).
\]

In order to do this, use

\[
p \left( y_t | \pi^{(i)}_{t|t-1}, \xi^{(i)}_{t|t-1} \right) = N \left( h_{0,t} + h_{\pi, t}\pi^{(i)}_{t|t-1} + H_t^i\xi^{(i)}_{t|t-1}, H_t^iP_{t|t-1}H_t + R_t \right)
\]

so that

\[
q_t^{(i)} = w_{t-1}^{(i)} \times \left| \Omega_t \right|^{-1/2} \times \exp \left\{ -\frac{1}{2} \left( y_t - h_{0,t} - h_{\pi, t}\pi^{(i)}_{t|t-1} - H_t^i\xi^{(i)}_{t|t-1} \right)' \times \Omega_t^{-1} \times \left( y_t - h_{0,t} - h_{\pi, t}\pi^{(i)}_{t|t-1} - H_t^i\xi^{(i)}_{t|t-1} \right) \right\}.
\]

where \( w_{t-1}^{(i)} \) denotes the particle weight from the previous period. (In the expression above we eliminate the constant coefficient that is independent of \( i \) and the model parameters.)

3. **Re-sampling.**\(^{44}\) Provided the number of effective particles (effective sample size), computed as

\[
ESS = \frac{1}{\sum \left( w_t^{(i)} \right)^2},
\]

\(^{44}\)This means (roughly speaking) increasing the number of particles receiving high weight \( \left( \frac{q_t^{(i)}}{\sum q_t^{(j)}} \right) \) and eliminating particles with very low weight, while maintaining the number of particles to \( N \).
falls below the threshold ($ESS < 0.75 * N$) we re-sample such that

\[ p \left( \left[ \hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)} \right] \mid \left[ \hat{\pi}_{t|t-1}^{(j)}, k_{t|t-1}^{(j)} \right] \right) = \frac{q_{t|t}^{(j)}}{\sum q_{t|t}^{(j)}}. \]

Here we use **systematic resampling**: see Kitagawa (1996), Hol, Schönh, and Gustafsson (2006) for a discussion of resampling and different methods. The outcome of systematic resampling is a discrete distribution with particles \( \left\{ \hat{\pi}_{t|t}^{(k)}, k_{t|t}^{(k)} \right\}_{k=1}^{N} \) and corresponding weights \( w_t(i) = 1/N \) for \( i = 1, \ldots, N \). In case of not resampling the weights are \( w_t(i) = q_{t|t}^{(i)} / \sum q_{t|t}^{(i)}. \)

**4. Linear measurement equation:** for \( i = 1, \ldots, N \), evaluate

\[
\xi_{t|t}^{(i)} = \xi_{t|t-1}^{(i)} + K_t \left( y_t - h_{0,t} - h_{\pi,t} \hat{\pi}_{t|t}^{(i)} - H_t \xi_{t|t-1}^{(i)} \right)
\]

\[
K_t = P_{t|t-1} H_t \Omega_t^{-1}
\]

\[
P_{t|t} = P_{t|t-1} - K_t H'_t P_{t|t-1}
\]

**5. Particle filter prediction.** For \( i = 1, \ldots, N \), compute

\[
k_{t+1|t}^{(i)} = f_k(\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)})
\]

and then draw \( \hat{\pi}_{t+1|t}^{(i)} \) from distribution

\[
p \left( \hat{\pi}_{t+1|t}^{(i)} \mid Y_t, \hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)} \right)
\]

\[
= N \left( f_\pi(\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}) + f_k(\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)})^{-1} \xi_{t|t}^{(i)}; \ k_k(\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)})^{-2} P_{t|t}^{[\pi,\eta]} \right)
\]

where we use the notation: \( P_{t|t}^{[x,z]} = P_{t|t} (x, z) \).

**6. Linear model prediction**

\[
\xi_{t+1|t}^{(i)} = \xi_{t|t}^{(i)} + \tilde{K}_t^{(i)} \left( \hat{\pi}_{t+1|t}^{(i)} - f_\pi(\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}) - f_k(\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)})^{-1} \xi_{t|t}^{(i)} \right)
\]

\[
\xi_{t+1|t}^{(i)} = f_\xi(\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}) + A_\xi (\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}) \xi_{t|t}^{(i)}
\]

\[
P_{t+1|t} = Q_\xi + \tilde{P}_{t|t}; \ Q_\xi = S_\xi \Sigma S_\xi';
\]

where

\[
\tilde{K}_t^{(i)} = P_{t|t} A'_\pi \left( \hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)} \right) \left( A_\pi (\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}) P_{t|t} A'_\pi (\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}) \right)^{-1}
\]

\[
= f_k(\hat{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}) \left[ \begin{array}{c}
\frac{1}{P_{t|t}^{[\pi,\eta]} / P_{t|t}^{[\pi,\eta]}}, \\
\frac{P_{t|t}^{[\pi,\eta]} / P_{t|t}^{[\pi,\eta]} \cdot P_{t|t}^{[\pi,\eta]} / P_{t|t}^{[\pi,\eta]}}{P_{t|t}^{[\pi,\eta]} / P_{t|t}^{[\pi,\eta]} \cdot P_{t|t}^{[\pi,\eta]} / P_{t|t}^{[\pi,\eta]}} \end{array} \right];
\]

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Anchored Inflation Expectations

and

\[ A_t(\pi_{t|t}^{(i)}, k_{t|t}^{(i)}) \tilde{K}^{(i)}_t = f_k(\pi_{t|t}^{(i)}, k_{t|t}^{(i)}) \begin{bmatrix} 0 \\ \frac{p_{t|t}}{p_{t|t}} \rho \phi + \frac{p_{t|t}}{p_{t|t}} \rho \phi + f_k(\pi_{t|t}^{(i)}, k_{t|t}^{(i)})^{-1} (1 - \gamma) \Gamma \end{bmatrix} \]

\[ \tilde{P}_{t|t} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_1 & 0 \\ 0 & \rho_2 + \rho_1 & \rho_2 \end{bmatrix} \]

where

\[ \rho_1 = -\left( \frac{p_{t|t}}{p_{t|t}} \right)^2 \rho_\phi, \]

\[ \rho_2 = -\left( \frac{p_{t|t}}{p_{t|t}} \right)^2 \rho_\phi, \]

Notice that, importantly, \( P_{t+1|t} \) is independent of particles. This is key for a fast evaluation of the likelihood.

Finally, the log-Likelihood is approximated by

\[ L(\cdot) = \sum_{t=1}^{T} \log p(y_t|Y_{t-1}) \]

where

\[ p(y_t|Y_{t-1}) = p(y_t|\xi_t, [\pi_t, k_t]) p(\xi_t, [\pi_t, k_t]|Y_{t-1}) \]

\[ = p(y_t|\xi_t, [\pi_t, k_t]) p(\xi_t|[\pi_t, k_t], Y_{t-1}) p([\pi_t, k_t]|Y_{t-1}), \]

\[ \rightarrow \]

\[ L(\cdot) \approx \sum_{t=1}^{T} \log \left( \sum_{i=1}^{N} q_t^{(i)} \right). \]

Algorithm ends.

In the estimation performed in this paper we set the number of particles \( N = 2500; \) To avoid injecting randomness in the calculation of the likelihood, the “chatter” of changing random numbers, we keep the simulated (standardized) innovations constant as we evaluate different parameters—see the discussion in Fernandez-Villaverde and Rubio-Ramirez (2007).

In detail, we fix the following innovations: random initial conditions for the nonlinear state variables; random draws to compute shocks in the nonlinear prediction step; and random draws in the resampling step.
\section*{B Marginalized Smoother}

We follow Lindsten and Schön (2013) using the ‘joint backward simulation’ Rao-Blackwellised particle smoother. See also Godsill, Doucet, and West (2004). We compute a smoothed path for the states, conditional on a parameter draw, for the sample \( t = 1 \ldots T \). The algorithm, in conjunction with the forward filter above, allows to produce the full distribution of state and parameters using Carter and Khon (1994).

The objective is to draw \( j = 1 \ldots M \) trajectories of the model variables \( \left\{ \tilde{\pi}_{t|T}^{(j)}, \tilde{\kappa}_{t|T}^{(j)}, \tilde{\xi}_{t|T}^{(j)} \right\}_{t=1}^{T} \). The forward filter allows to draw \( \tilde{\pi}_{t|T}^{(j)} \tilde{\kappa}_{T|T}^{(j)} \) from the empirical distribution of \( \left\{ \tilde{\pi}_{t|t}^{(k)}, \tilde{\kappa}_{t|t}^{(k)} \right\}_{k=1}^{N} \) where each particle has weight \( w_{t}^{(k)} \). Moreover, conditional on the draw \( \tilde{\pi}_{T|T}^{(j)}, \tilde{\kappa}_{T|T}^{(j)} \), it allows to draw the linear state \( \tilde{\xi}_{t|T}^{(j)} \) from the normal distribution \( N \left( \tilde{\xi}_{T|T}^{(j)}, P_{T|T} \right) \). Given this we are computing \( M \) paths as follows.

\textbf{Algorithm:}

For \( t = T - 1 : -1 : 1 \)

For each \( j = 1 \ldots M \)

For each \( i = 1 \ldots N \), compute

\[
 w_{t|t+1}^{j}(i) = \frac{w_{t}(i)p \left( \tilde{\pi}_{t+1|t+1}^{(j)}, \tilde{\kappa}_{t+1|t+1}^{(j)}, \tilde{\xi}_{t+1|t+1}^{(j)} | \tilde{\pi}_{t|t}^{(i)}, \tilde{\kappa}_{t|t}^{(i)}, Y_{t} \right)}{\sum_{k}^{N} w_{t}(k)p \left( \tilde{\pi}_{t+1|t+1}^{(j)}, \tilde{\kappa}_{t+1|t+1}^{(j)}, \tilde{\xi}_{t+1|t+1}^{(j)} | \tilde{\pi}_{t|t}^{(k)}, \tilde{\kappa}_{t|t}^{(k)}, Y_{t} \right)}
\]

where the last line makes use of the fact that \( w_{t}(i) = 1/N \) because of resampling in the forward filter. The probability distribution above can be expressed as

\[
 p \left( \tilde{\pi}_{t+1|T}, \kappa_{t+1|T}, \xi_{t+1|T} | \tilde{\pi}_{t|t}^{(i)}, \kappa_{t|t}^{(i)}, Y_{t} \right) = p \left( \kappa_{t+1|T} | \tilde{\pi}_{t|t}^{(i)}, \kappa_{t|t}^{(i)}, Y_{t} \right) \times p \left( \xi_{t+1|T} | \tilde{\pi}_{t|t}^{(i)}, \kappa_{t|t}^{(i)}, Y_{t} \right)
\]

which uses \( \kappa_{t+1|t} = f_{k}(\tilde{\pi}_{t|t}^{(i)}, \kappa_{t|t}^{(i)}) \). We can then evaluate

\[
 p \left( \tilde{\pi}_{t+1|T}, \tilde{\xi}_{t+1|T} | \tilde{\pi}_{t|t}^{(i)}, \kappa_{t|t}^{(i)}, Y_{t} \right) \times 1_{\tilde{\kappa}_{t+1}^{(j)} = f_{k}(\tilde{\pi}_{t|t}^{(i)}, \kappa_{t|t}^{(i)})} = \begin{cases} 
 p_{t|t+1}^{j,i}, & \text{if } \tilde{\kappa}_{t+1|T} = f_{k}(\tilde{\pi}_{t|t}^{(i)}, \kappa_{t|t}^{(i)}) \\
 0, & \text{otherwise}
\end{cases}
\]
where

\[ p^{j,i}_t \leftarrow \alpha \exp \left\{ -\frac{1}{2} \ln \left( \left| \hat{\Omega}_t^{(i)} \right| \right) - \frac{1}{2} \left( \eta^{j,i}_t \right)^\prime \left( \hat{\Omega}_t^{(i)} \right)^{-1} \left( \eta^{j,i}_t \right) \right\} \]

\[ \eta^{j,i}_t = \left[ \pi^{(j)}_t \left( \bar{q}^{(j)}_t, \tilde{q}^{(j)}_t \right) \right] - \left[ f \left( \hat{\pi}_t^{(i)}, k_t^{(i)} \right) + A \left( \hat{\pi}_t^{(i)}, k_t^{(i)} \right) \xi_t^{(i)} \right]. \]

\[ \tilde{\Omega}_t^{(i)} = Q + A \left( \hat{\pi}_t^{(i)}, k_t^{(i)} \right) P_t A \left( \hat{\pi}_t^{(i)}, k_t^{(i)} \right)^\prime \]

\[ Q = \begin{bmatrix} 0 & 0 \\ 0 & S_x \Sigma S_x^\prime \end{bmatrix}. \]

and where we can use

\[ 1_{k_t^{(i)} = k_t^{(j)}} = \begin{cases} 1, & \text{if } k_t^{(i)} = k_t^{(j)} \\ 0, & \text{otherwise} \end{cases} \]

\[ = \begin{cases} 1, & \text{if } \left| (1-\gamma)(1-\gamma^{-1}) \hat{\pi}_t^{(i)} \right| > \sqrt{\text{MSE}} \\ 0, & \text{otherwise} \end{cases} \]

\[ + \left( 1 - 1_{k_t^{(i)} = k_t^{(j)}} \right) \times \left( 1, \left| (1-\gamma)(1-\gamma^{-1}) \hat{\pi}_t^{(i)} \right| \leq \sqrt{\text{MSE}} \right) \times 1_{k_t^{(i)} = k_t^{(j)} - 1}. \]

Moving to the linear variables \( \xi_t \), for each \( j = 1...M \), draw the nonlinear variables \( \tilde{\pi}_t^{(j)}(T), k_t^{(j)}(T) \) from \( \left\{ \pi_t^{(k)}(T), k_t^{(k)}(T) \right\}_{k=1}^N \) using the new set of weights \( \left\{ w_t^j(k) \right\}_{k=1}^N \). Conditional on the draw, sample from

\[ p \left( \xi_t | \tilde{\pi}_t^{(j)}, k_t^{(j)}, \tilde{\pi}_t^{(j)+1}, \tilde{\pi}_t^{(j)+1}, Y_t \right). \]

In particular we draw \( \tilde{\xi}_t^{(j)}(T) \) from the distribution

\[ N \left( \xi_t^{(j)} + \Delta_t^{(j)} \left( \hat{\pi}_t^{(j)} + \hat{\xi}_t^{(j)}(T) \right)^\prime f \left( \hat{\pi}_t^{(j)}(T), \xi_t^{(j)}(T) \right) - A \left( \hat{\pi}_t^{(j)}(T), \hat{\xi}_t^{(j)}(T) \right) \xi_t^{(j)}(T), A_t^{(j)} \right) \]

where \( \xi_t^{(j)} \) is the element in \( \left\{ \pi_t^{(k)}(T), k_t^{(k)}(T) \right\}_{k=1}^N \) that corresponds to the same draw \( j \) from which the particles \( \tilde{\pi}_t^{(j)}(T), k_t^{(j)}(T) \) are obtained, and where

\[ \Delta_t^{(j)} = P_t A \left( \hat{\pi}_t^{(j)}(T), \hat{k}_t^{(j)}(T) \right)^\prime \left( Q + A \left( \hat{\pi}_t^{(j)}(T), k_t^{(j)}(T) \right) P_t A \left( \hat{\pi}_t^{(j)}(T), k_t^{(j)}(T) \right)^\prime \right)^{-1} \]

\[ A_t^{(j)} = P_t - \Delta_t^{(j)} A \left( \hat{\pi}_t^{(j)}(T), \hat{k}_t^{(j)}(T) \right) P_t. \]

Algorithm ends.
C Estimation in other countries

Observation equation. The Consensus forecasts can be expressed as

\[ E_t^{Cons} \pi_{Y1, Q2} = \sum_{j=1}^{4} w(j) \pi_{t-J} + \hat{E}_t \sum_{i=0}^{2} w(5+i) \pi_{t+i} \]

\[ E_t^{Cons} \pi_{Y1, Q1} = \sum_{j=1}^{3} w(j) \pi_{t-J} + \hat{E}_t \sum_{i=0}^{3} w(4+i) \pi_{t+i} \]

where the vector

\[ w = \left( \frac{1}{16}, \frac{2}{16}, \frac{3}{16}, \frac{4}{16}, \frac{3}{16}, \frac{2}{16}, \frac{1}{16} \right) \]

defines the appropriate weights, and the notation \( \pi_{i,j} \) denotes the inflation forecast of year \( i \) inflation, taken in the current year in quarter \( j \). The remaining four forecasts concern expectations for inflation in the next calendar year, taken in each quarter of the current year. Similarly they can be expressed

\[ E_t^{Cons} \pi_{Y2, Q4} = \sum_{j=1}^{2} w(j) \pi_{t-J} + \hat{E}_t \sum_{i=0}^{4} w(3+i) \pi_{t+i} \]

\[ E_t^{Cons} \pi_{Y2, Q3} = \sum_{j=1}^{1} w(j) \pi_{t-J} + \hat{E}_t \sum_{i=0}^{5} w(2+i) \pi_{t+i} \]

\[ E_t^{Cons} \pi_{Y2, Q2} = \hat{E}_t \sum_{i=0}^{6} w(1+j) \pi_{t+i} \]

\[ E_t^{Cons} \pi_{Y2, Q1} = \hat{E}_t \sum_{i=1}^{7} w(j) \pi_{t+i} \]

where the last two forecasts are purely forward looking. The observation equation can then be written

\[
\begin{bmatrix}
\hat{E}_t \sum_{i=1}^{2} w(5+i) \pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{3} w(4+i) \pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{4} w(3+i) \pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{5} w(2+i) \pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{6} w(1+i) \pi_{t+i} \\
\hat{E}_t \sum_{i=1}^{7} w(i) \pi_{t+i}
\end{bmatrix} = \pi^{*, F} + H_t \xi_t + R_t \omega_t^{Cons},
\]

where \( \pi^{*, F} \) denotes the country-specific sample mean of inflation.

As discussed in the main text, the aim is to evaluate model predictions under the posterior distribution obtained with US data on inflation and forecasts by professional forecasters. However, there are few parameters that we choose to estimate independently. In particular,
we estimate the inflation mean and the standard deviation of measurement error on survey-forecasts. These parameters are necessarily country-specific and can impact significantly the model’s predictions.

For the US, the Metropolis-Hasting algorithm is used to simulate the posterior distribution

\[ P(\theta^US|Y^US_t) = L(Y^US_t|\theta^US)P(\theta^US) \]

where \( L(Y^US_t|\theta^US) \) the model Likelihood. For the other countries we use the US posterior distribution as prior for the common parameters. We can then simulate the posterior distribution

\[ P^F(\theta^F|Y^F_t,Y^US_t,\theta^US) = \bar{L}(Y^F_t|\theta^US,\theta^F)\lambda^F L(Y^US_t|\theta^US)p(\theta^US)p(\theta^F) \]

where the parameter \( \lambda^F \) is the weight that is given to the likelihood of the model for other country. Notice that the case of \( \lambda^F = 0 \) corresponds to evaluating the parameters that are common to the US, \( \theta^US \), at the posterior distribution for the US. The remaining parameters, \( \theta^F \), are instead evaluated at their prior. In our estimation we set \( \lambda^F = 0.05 \) implying a very low weight on the foreign model likelihood, \( \bar{L}(Y^F_t|\theta^US,\theta^F) \). As a result, the posterior distribution of the common parameters with the US is essentially the same as for the US, while the likelihood is used informs about the country-specific parameters. Tables in the additional technical appendix give the parameter estimates for all other countries. They are obtained using 200000 draws from the simulated posterior distribution.

D Marginal Likelihood

To compute the marginal likelihood for the US baseline model we use the Geweke harmonic mean estimator. For each draw \( \theta_i \) we compute

\[ p(y) = \left\{ \frac{1}{D} \sum_{i=1}^{D} \frac{f(\theta_i)}{p(y|\theta_i) p(\theta_i)} \right\}^{-1} \]

where the function \( f(\cdot) \) is the density of a Normal distribution with mean and variance corresponding to the mean and variance of the posterior draws sample. Moreover the distribution is truncated so that

\[ f(\theta_i) = \tau^{-1} (2\pi)^{-d/2} |V_\theta| \exp\left[-0.5 (\theta_i - \bar{\theta})^\prime V_\theta^{-1} (\theta_i - \bar{\theta})\right] \times \left\{ \theta_i : (\theta_i - \bar{\theta})^\prime V_\theta^{-1} (\theta_i - \bar{\theta}) < \chi^2_{\tau,d} \right\} \]

where \( \chi^2_{\tau,d} \) is the \((1 - \tau)\) quantile of the \( \chi^2_d \) distribution and \( d \) is the dimension of the parameters’ vector. In order to compute the marginal likelihood we set \( \tau = 0.5 \).
APPENDIX: ADDITIONAL MATERIAL (NOT FOR PUBLICATION)

D.0.1 Derivation and Proofs. The crucial step is the derivation of the prediction for the linear state (step 6). Notice first that given the link between \( \bar{\pi}_t \) and the linear state we can use

\[
\begin{bmatrix}
\xi_t | Y_t, \bar{\pi}_{t|t}, k_{t|t}^{(i)} \\
\bar{\pi}_{t+1|t} - f_{\pi}(\bar{\pi}_{t|t}, k_{t|t}^{(i)}) | Y_t, \bar{\pi}_{t|t}, k_{t|t}^{(i)}
\end{bmatrix} 
\sim N \left( \begin{bmatrix}
\xi_{t|t}^{(i)} \\
A_{\bar{\pi},\xi|t}^{(i)}
\end{bmatrix}, \begin{bmatrix}
P_{t|t}^{(i)} A_{\bar{\pi},\pi|t}^{(i)} & A_{\bar{\pi},\pi|t}^{(i)} P_{t|t}^{(i)} \\
P_{t|t}^{(i)} A_{\bar{\pi},\pi|t}^{(i)} & A_{\bar{\pi},\pi|t}^{(i)} P_{t|t}^{(i)}
\end{bmatrix} \right)
\]

Using properties of the normal distribution, we can now get the conditional distribution

\[
\begin{align*}
\xi_{t|t}^{(i)} &= \xi_{t|t}^{(i)} + P_{t|t}^{(i)} A_{\bar{\pi},\xi|t}^{(i)} \left( A_{\bar{\pi},\pi|t}^{(i)} P_{t|t}^{(i)} A_{\bar{\pi},\pi|t}^{(i)} \right)^{-1} \left( \bar{\pi}_{t+1|t} - f_{\pi}(\bar{\pi}_{t|t}, k_{t|t}^{(i)}) - f_{k,t}^{(i)} - 1 \xi_{t|t}^{(i)} \right) \\
P_{t|t}^{(i)} &= P_{t|t}^{(i)} - P_{t|t}^{(i)} A_{\bar{\pi},\pi|t}^{(i)} \left( A_{\bar{\pi},\pi|t}^{(i)} P_{t|t}^{(i)} A_{\bar{\pi},\pi|t}^{(i)} \right)^{-1} A_{\bar{\pi},\pi|t}^{(i)} P_{t|t}^{(i)}
\end{align*}
\]

where

\[
A_{\bar{\pi},\xi|t}^{(i)} = A_{\xi}(\bar{\pi}_{t|t}, k_{t|t}^{(i)}); \\
f_{k,t}^{(i)} = f_{k}(\bar{\pi}_{t|t}, k_{t|t}^{(i)})^{-1}.
\]

The predictions for the linear state are then

\[
\xi_{t+1|t}^{(i)} = f_{\xi,t}^{(i)} + A_{\xi,t}^{(i)} \xi_{t|t}^{(i)}
\]

where

\[
f_{\xi,t}^{(i)} = f_{\xi}(\bar{\pi}_{t|t}, k_{t|t}^{(i)});
\]

\[
A_{\xi,t}^{(i)} = A_{\xi}(\bar{\pi}_{t|t}, k_{t|t}^{(i)});
\]

and

\[
P_{t+1|t}^{(i)} = A_{\xi,t}^{(i)} P_{t|t}^{(i)} A_{\xi,t}^{(i)} + \]

\[
-A_{\xi,t}^{(i)} \left[ P_{t|t}^{(i)} A_{\bar{\pi},\pi|t}^{(i)} \left( A_{\bar{\pi},\pi|t}^{(i)} P_{t|t}^{(i)} A_{\bar{\pi},\pi|t}^{(i)} \right)^{-1} A_{\bar{\pi},\pi|t}^{(i)} P_{t|t}^{(i)} \right] A_{\xi,t}^{(i)} + Q_{\xi}.
\]

Here we show that the \( P_{t+1|t}^{(i)} = P_{t+1|t} \) for every \( (i). \) For given initial \( P_{t|t} \), it is straightforward to show that

\[
\left( A_{\bar{\pi},\pi|t}^{(i)} P_{t|t}^{(i)} A_{\bar{\pi},\pi|t}^{(i)} \right)^{-1} = \frac{1}{f_{k,t}^{(i)} - 2 P_{t|t}^{(i)} \Sigma_{k,t}^{(i)}},
\]

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Then a little algebra leads to the following:

\[
\tilde{P}^{(i)}_{t|t} = P_{t|t} A^{(i)}_{\bar{x},t} \left( \frac{1}{A^{(i)}_{\bar{x},t} P_{t|t}} \right) A^{(i)}_{\bar{x},t} P_{t|t}
\]

\[
= \begin{bmatrix}
P_{t|t}^{[\eta,\eta]} & P_{t|t}^{[\eta,\varphi]} & P_{t|t}^{[\eta,\pi]} \\
& P_{t|t}^{[\eta,\varphi]} && P_{t|t}^{[\eta,\pi]} \\
& & P_{t|t}^{[\eta,\pi]} 
\end{bmatrix}
\]

\[
= \tilde{P}_{t|t}.
\]

Next, evaluate

\[
\tilde{P}_{t|t} = A^{(i)}_{\bar{x},t} P_{t|t} A^{(i)}_{\bar{x},t} - A^{(i)}_{\bar{x},t} \tilde{P}_{t|t} A^{(i)}_{\bar{x},t} = A^{(i)}_{\bar{x},t} \left( P_{t|t} - \tilde{P}_{t|t} \right) A^{(i)}_{\bar{x},t}
\]

where

\[
(P_{t|t} - \tilde{P}_{t|t}) = \begin{bmatrix}
0 & 0 & 0 \\
0 & P_{t|t}^{[\varphi,\varphi]} - \left( \frac{P_{t|t}^{[\eta,\varphi]}}{P_{t|t}^{[\eta,\eta]}} \right)^2 & P_{t|t}^{[\varphi,\pi]} - \frac{\left( \frac{P_{t|t}^{[\eta,\varphi]}}{P_{t|t}^{[\eta,\eta]}} \right)^2 P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} \\
0 & P_{t|t}^{[\eta,\varphi]} - \left( \frac{P_{t|t}^{[\eta,\varphi]}}{P_{t|t}^{[\eta,\eta]}} \right)^2 \frac{P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} & P_{t|t}^{[\pi,\pi]} - \frac{\left( \frac{P_{t|t}^{[\eta,\varphi]}}{P_{t|t}^{[\eta,\eta]}} \right)^2 P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}}
\end{bmatrix}.
\]

Finally,

\[
\tilde{P}_{t|t} = A^{(i)}_{\bar{x},t} \left( P_{t|t} - \tilde{P}_{t|t} \right) A^{(i)}_{\bar{x},t} = \begin{bmatrix}
0 & 0 & 0 \\
0 & \ell_1 & \ell_2 + \ell_1 \\
0 & \ell_2 + \ell_1 & 2\ell_2 + \ell_1 + \left( P_{t|t}^{[\pi,\pi]} - \left( \frac{P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} \right)^2 \right) \gamma^2
\end{bmatrix}
\]

where

\[
\ell_1 = \rho_{\varphi}^2 \left( P_{t|t}^{[\varphi,\varphi]} - \left( \frac{P_{t|t}^{[\eta,\varphi]}}{P_{t|t}^{[\eta,\eta]}} \right)^2 \right)
\]

\[
\ell_2 = \left( - \frac{P_{t|t}^{[\eta,\varphi]} P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} + P_{t|t}^{[\varphi,\pi]} \right) \rho_{\varphi} \gamma.
\]

So we can express

\[
P^{(i)}_{t+1|t} = P_{t+1|t} = Q_{\xi} + \tilde{P}_{t|t}.
\]